

Math 1A Fall 2001: Section 2.3 Solutions

2. (a) $\lim_{x \rightarrow 2} [f(x) + g(x)] = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 2 + 0 = 2$

(b) $\lim_{x \rightarrow 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x \rightarrow 0} [f(x)g(x)] = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) = 0 \cdot 1.3 = 0$

(d) Since $\lim_{x \rightarrow -1} g(x) = 0$ and g is in the denominator, the given limit does not exist.

(e) $\lim_{x \rightarrow 2} x^3 f(x) = \left[\lim_{x \rightarrow 2} x^3 \right] \left[\lim_{x \rightarrow 2} f(x) \right] = 2^3 \cdot 2 = 16$

(f) $\lim_{x \rightarrow 1} \sqrt{3 + f(x)} = \sqrt{3 + \lim_{x \rightarrow 1} f(x)} = \sqrt{3 + 1} = 2$

4. $\lim_{x \rightarrow 2} \frac{2x^2 + 1}{x^2 + 6x - 4} = \frac{\lim_{x \rightarrow 2} (2x^2 + 1)}{\lim_{x \rightarrow 2} (x^2 + 6x - 4)}$ (Limit Law 5)

$= \frac{2 \lim_{x \rightarrow 2} x^2 + \lim_{x \rightarrow 2} 1}{\lim_{x \rightarrow 2} x^2 + 6 \lim_{x \rightarrow 2} x - \lim_{x \rightarrow 2} 4}$ (2, 1, & 3)

$= \frac{2(2)^2 + 1}{(2)^2 + 6(2) - 4} = \frac{9}{12} = \frac{3}{4}$ (9, 7, & 8)

8. (a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \rightarrow 2$, just as in Example 3. Remember that in finding $\lim_{x \rightarrow a} f(x)$, we never consider $x = a$.

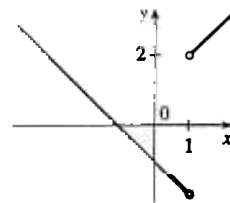
12. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x+1} = \frac{1^2 + 1 + 1}{1 + 1} = \frac{3}{2}$

16. $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2} = \lim_{x \rightarrow 2} \frac{(x+2)(x-2)(x^2+4)}{x-2} = \lim_{x \rightarrow 2} (x+2)(x^2+4) = \lim_{x \rightarrow 2} (x+2) \lim_{x \rightarrow 2} (x^2+4)$
 $= (2+2)(2^2+4) = 32$

20. $\lim_{t \rightarrow 0} \left[\frac{1}{t} - \frac{1}{t^2 + t} \right] = \lim_{t \rightarrow 0} \frac{(t^2 + t) - t}{t(t^2 + t)} = \lim_{t \rightarrow 0} \frac{t^2}{t \cdot t(t+1)} = \lim_{t \rightarrow 0} \frac{1}{t+1} = \frac{1}{0+1} = 1$

34. (a) (i) $\lim_{x \rightarrow 1^+} \frac{x^2 - 1}{|x - 1|} = \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1^+} (x + 1) = 2$ (c)

(ii) $\lim_{x \rightarrow 1^-} \frac{x^2 - 1}{|x - 1|} = \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{-(x - 1)} = \lim_{x \rightarrow 1^-} -(x + 1) = -2$



(b) No, $\lim_{x \rightarrow 1} F(x)$ does not exist since $\lim_{x \rightarrow 1^+} F(x) \neq \lim_{x \rightarrow 1^-} F(x)$.

44. **Solution 1:** First, we find the coordinates of P and Q as functions of r . Then we can find the equation of the line determined by these two points, and thus find the x -intercept (the point R), and take the limit as $r \rightarrow 0$.

The coordinates of P are $(0, r)$. The point Q is the point of intersection of the two circles $x^2 + y^2 = r^2$ and $(x - 1)^2 + y^2 = 1$. Eliminating y from these equations, we get $r^2 - x^2 = 1 - (x - 1)^2 \Leftrightarrow r^2 = 1 + 2x - 1 \Leftrightarrow x = \frac{1}{2}r^2$. Substituting back into the equation of the shrinking circle to find the y -coordinate, we get

$(\frac{1}{2}r^2)^2 + y^2 = r^2 \Leftrightarrow y^2 = r^2(1 - \frac{1}{4}r^2) \Leftrightarrow y = r\sqrt{1 - \frac{1}{4}r^2}$ (the positive y -value). So the coordinates

of Q are $(\frac{1}{2}r^2, r\sqrt{1 - \frac{1}{4}r^2})$. The equation of the line joining P and Q is thus

$y - r = \frac{r\sqrt{1 - \frac{1}{4}r^2} - r}{\frac{1}{2}r^2 - 0} (x - 0)$. We set $y = 0$ in order to find the x -intercept, and get

$x = -r \frac{\frac{1}{2}r^2}{r(\sqrt{1 - \frac{1}{4}r^2} - 1)} = \frac{-\frac{1}{2}r^2(\sqrt{1 - \frac{1}{4}r^2} + 1)}{1 - \frac{1}{4}r^2 - 1} = 2(\sqrt{1 - \frac{1}{4}r^2} + 1)$

Now we take the limit as $r \rightarrow 0^+$: $\lim_{r \rightarrow 0^+} x = \lim_{r \rightarrow 0^+} 2(\sqrt{1 - \frac{1}{4}r^2} + 1) = \lim_{r \rightarrow 0^+} 2(\sqrt{1} + 1) = 4$.

So the limiting position of R is the point $(4, 0)$.