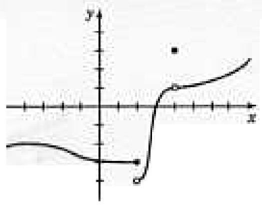
# Math 1A Fall 2001: Section 2.4 Solutions

**4.** $g$ is continuous on $[-4,-2)$, $(-2,2)$, $[2,4)$, $(4,6)$, and $(6,8)$.

**6.**




**8.** (a) Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.

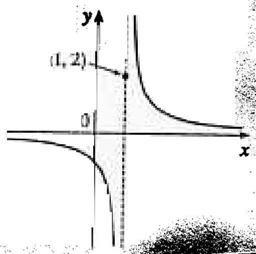
(b) Continuous; the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.

(c) Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values — at a cliff, for example.

(d) Discontinuous; as the distance traveled increases, the cost of the ride jumps in small increments.

(e) Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

**14.** $f(x) = \begin{cases} 1/(x-1) & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ is discontinuous at 1 because $\lim_{x\to 1} f(x)$ does not exist.




**18.** $G(t) = 25 - t^2$ is a polynomial, so it is continuous (Theorem 5). $F(x) = \sqrt{x}$ is continuous by Theorem 7. So, by Theorem 9, $F(G(t)) = \sqrt{25-t^2}$ is continuous on its domain, which is $\{t \mid 25 - t^2 \geq 0\} = \{t \mid |t| \leq 5\} = [-5, 5]$. Also, $2t$ is continuous on $\mathbb{R}$, so by Theorem 4 #1, $f(t) = 2t + \sqrt{25-t^2}$ is continuous on its domain, which is $[-5, 5]$.

**28.** Because arctan is a continuous function, we can apply Theorem 8.

$$\lim_{x\to 2} \arctan\left(\frac{x^2-4}{3x^2-6x}\right) = \arctan\left(\lim_{x\to 2}\frac{(x+2)(x-2)}{3x(x-2)}\right) = \arctan\left(\lim_{x\to 2}\frac{x+2}{3x}\right) = \arctan\tfrac{2}{3} \approx 0.588$$

**38.** $f(x) = \ln x - e^{-x}$ is continuous on the interval $[1,2]$, $f(1) = -e^{-1} \approx -0.37$, and $f(2) = \ln 2 - e^{-2} \approx 0.56$. Since $-0.37 < 0 < 0.56$, there is a number $c$ in $(1,2)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\ln x - e^{-x} = 0$, or $\ln x = e^{-x}$, in the interval $(1,2)$.

**40.** (a) $f(x) = x^5 - x^2 + 2x + 3$ is continuous on $[-1, 0]$, $f(-1) = -1 < 0$, and $f(0) = 3 > 0$. Since $-1 < 0 < 3$, there is a number $c$ in $(-1, 0)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $x^5 - x^2 + 2x + 3 = 0$ in the interval $(-1, 0)$.

(b) $f(-0.88) \approx -0.062 < 0$ and $f(-0.87) \approx 0.0047 > 0$, so there is a root between $-0.88$ and $-0.87$.