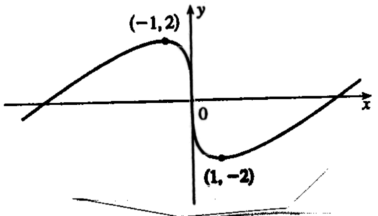
# Math 1A Fall 2001: Section 4.3 Solutions PART 2

**20.** (a) $Q(x) = x - 3x^{1/3} \Rightarrow Q'(x) = 1 - \frac{1}{x^{2/3}} > 0 \Leftrightarrow x^{2/3} > 1 \Leftrightarrow x^2 > 1 \Leftrightarrow x < -1$ or $x > 1$, so $Q$ is increasing on $(-\infty, -1)$, and $(1, \infty)$, and decreasing on $(-1, 1)$.

(b) $Q'(x) = 0 \Leftrightarrow x = \pm 1$; $Q(1) = -2$ is a local minimum, and $Q(-1) = 2$ is a local maximum.

(c) $Q''(x) = \frac{2}{3}x^{-5/3} > 0 \Leftrightarrow x > 0$, so $Q$ is CU on $(0, \infty)$ and CD on $(-\infty, 0)$. Inflection point at $(0, 0)$.

(d)

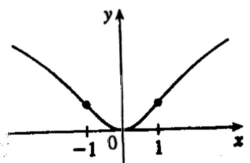



**24.** (a) $f(x) = \ln(1 + x^2) \Rightarrow f'(x) = \frac{2x}{1 + x^2} > 0 \Leftrightarrow x > 0$, so $f$ is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(b) $f(0) = 0$ is a local minimum.

(c) $f''(x) = \frac{(1 + x^2)(2) - 2x(2x)}{(1 + x^2)^2} = \frac{2(1 - x^2)}{(1 + x^2)^2} > 0 \Leftrightarrow 1 - x^2 > 0 \Leftrightarrow |x| < 1$, so $f$ is CU on $(-1, 1)$, CD on $(-\infty, -1)$ and $(1, \infty)$. There are IP at $(1, \ln 2)$ and $(-1, \ln 2)$.

(d)



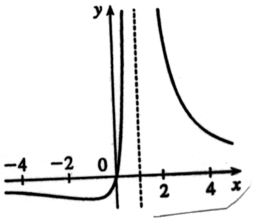

**26.** (a) $\lim_{x \to \pm\infty} \frac{x}{(x - 1)^2} = 0$, so $y = 0$ is a HA. $\lim_{x \to 1} \frac{x}{(x - 1)^2} = \infty$, so $x = 1$ is a VA.

(b) $f(x) = \frac{x}{(x - 1)^2} \Rightarrow f'(x) = \frac{(x - 1)^2(1) - x(2)(x - 1)}{[(x - 1)^2]^2} = \frac{(x - 1)[(x - 1) - 2x]}{(x - 1)^4} = \frac{-x - 1}{(x - 1)^3} = 0 \Rightarrow x = -1$. $f'$ is negative on $(-\infty, -1)$ and $(1, \infty)$ and positive on $(-1, 1)$, so $f(x)$ is decreasing on $(-\infty, -1)$ and $(1, \infty)$ and increasing on $(-1, 1)$.

(c) Local minimum $f(-1) = -\frac{1}{4}$, no local maximum.

(d) $f''(x) = \frac{(x - 1)^3(-1) + (x + 1)(3)(x - 1)^2}{(x - 1)^6} = \frac{2(x + 2)}{(x - 1)^4}$. This is negative on $(-\infty, -2)$, and positive on $(-2, 1)$ and $(1, \infty)$. So $f$ is CD on $(-\infty, -2)$ and CU on $(-2, 1)$ and $(1, \infty)$. $f$ has an inflection point at $(-2, -\frac{2}{9})$.

(e)




**42.** $f(x) = axe^{bx^2} \Rightarrow f'(x) = a\left[xe^{bx^2} \cdot 2bx + e^{bx^2} \cdot 1\right] = ae^{bx^2}(2bx^2 + 1)$. For $f(2) = 1$ to be a maximum value, we must have $f'(2) = 0$. $f(2) = 1 \Rightarrow 1 = 2ae^{4b}$ and $f'(2) = 0 \Rightarrow 0 = (8b + 1)ae^{4b}$. So $8b + 1 = 0$ $[a \neq 0] \Rightarrow b = -\frac{1}{8}$ and now $1 = 2ae^{-1/2} \Rightarrow a = \sqrt{e}/2$.

**48.** Let $v(t)$ be the velocity of the car $t$ hours after 2:00 P.M. Then $\frac{v(1/6) - v(0)}{1/6 - 0} = \frac{50 - 30}{1/6} = 120$. By the Mean Value Theorem, there is a number $c$ such that $0 < c < \frac{1}{6}$ with $v'(c) = 120$. Since $v'(t)$ is the acceleration at time $t$, the acceleration $c$ hours after 2:00 P.M. is exactly 120 mi/h$^2$.