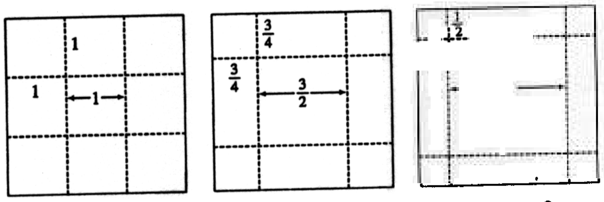
Math 1A Fall 2001: Section 4.6, Assignment 1 Solutions

2. The two numbers are $x+100$ and x. Minimize $f(x)=(x+100)x=x^2+100x$. $f'(x)=2x+100=0 \Rightarrow x=-50$. Since $f''(x)=2>0$, there is an absolute minimum at $x=-50$. The two numbers are 50 and -50.

4. Let $x>0$ and let $f(x)=x+1/x$. We wish to minimize $f(x)$. Now
$f'(x)=1-\frac{1}{x^2}=\frac{1}{x^2}(x^2-1)=\frac{1}{x^2}(x+1)(x-1)$, so the only critical number in $(0,\infty)$ is 1.
$f'(x)<0$ for $0<x<1$ and $f'(x)>0$ for $x>1$, so f has an absolute minimum at $x=1$, and $f(1)=2$.
Or: $f''(x)=2/x^3>0$ for all $x>0$, so f is concave upward everywhere and the critical point $(1,2)$ must correspond to a local minimum for f.

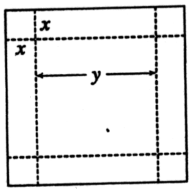
6. If the rectangle has dimensions x and y, then its area is $xy=1000\text{ m}^2$, so $y=1000/x$. The perimeter $P=2x+2y=2x+2000/x$. We wish to minimize the function $P(x)=2x+2000/x$ for $x>0$.
$P'(x)=2-2000/x^2=(2/x^2)(x^2-1000)$, so the only critical number in the domain of P is $x=\sqrt{1000}$.
$P''(x)=4000/x^3>0$, so P is concave upward throughout its domain and $P(\sqrt{1000})=4\sqrt{1000}$ is an absolute minimum value. The dimensions of the rectangle with minimal perimeter are $x=y=\sqrt{1000}=10\sqrt{10}$ m. (The rectangle is a square.)

8. (a)



The volumes of the resulting boxes are 1, 1.6875, and 2 ft^3. There appears to be a maximum volume of at least 2 ft^3.

(b) Let x denote the length of the side of the square being cut out. Let y denote the length of the base.



(c) Volume V = length × width × height $\Rightarrow V=y\cdot y\cdot x=xy^2$

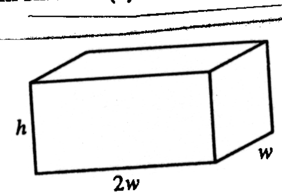
(d) Length of cardboard $=3 \Rightarrow x+y+x=3 \Rightarrow y+2x=3$

(e) $y+2x=3 \Rightarrow y=3-2x \Rightarrow V(x)=x(3-2x)^2$

(f) $V(x)=x(3-2x)^2 \Rightarrow$
$V'(x)=x\cdot 2(3-2x)(-2)+(3-2x)^2\cdot 1=(3-2x)[-4x+(3-2x)]=(3-2x)(-6x+3)$,
so the critical numbers are $x=\frac{3}{2}$ and $x=\frac{1}{2}$. Now $0\le x\le\frac{3}{2}$ and $V(0)=V(\frac{3}{2})=0$, so the maximum is $V(\frac{1}{2})=(\frac{1}{2})(2)^2=2\text{ ft}^3$, which is the value found from our third figure in part (a).

10. Let b be the length of the base of the box and h the height. The volume is $32{,}000=b^2h \Rightarrow h=32{,}000/b^2$. The surface area of the open box is $b^2+4hb=b^2+4(32{,}000/b^2)\,b=b^2+4(32{,}000)/b$. So $V'(b)=2b-4(32{,}000)/b^2=2(b^3-64{,}000)/b^2=0 \Leftrightarrow b=\sqrt[3]{64{,}000}=40$. This gives an absolute minimum since $V'(b)<0$ if $0<b<40$ and $V'(b)>0$ if $b>40$. The box should be $40\times40\times20$.

12.



$V=lwh \Rightarrow 10=(2w)(w)h=2w^2h$, so $h=5/w^2$. The cost is $10(2w^2)+6\,[2(2wh)+2(hw)]=20w^2+36wh$, so
$C(w)=20w^2+36w(5/w^2)=20w^2+180/w$.
$C'(w)=40w-180/w^2=40\left(w^3-\frac{9}{2}\right)/w^2 \Rightarrow w=\sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C when $w=\sqrt[3]{\frac{9}{2}}$ since $C'(w)<0$ for $0<w<\sqrt[3]{\frac{9}{2}}$ and $C'(w)>0$ for $w>\sqrt[3]{\frac{9}{2}}$. $C\left(\sqrt[3]{\frac{9}{2}}\right)=20\left(\sqrt[3]{\frac{9}{2}}\right)^2+\frac{180}{\sqrt[3]{9/2}}\approx \$163.54$.

14. The square of the distance from a point (x,y) on the parabola $x=-y^2$ is $x^2+(y+3)^2=y^4+y^2+6y+9=D(y)$. Now $D'(y)=4y^3+2y+6=2(y+1)(2y^2-2y+3)$. Since $2y^2-2y+3=0$ has no real roots, $y=-1$ is the only critical number. Then $x=-(-1)^2=-1$, so the point is $(-1,-1)$.