Math 1A Fall 2001: Section 4.9 Solutions

2. $f(x) = 1 - x^3 + 12x^5 \quad \Rightarrow \quad F(x) = x - \dfrac{x^{3+1}}{3+1} + 12\dfrac{x^{5+1}}{5+1} + C = x - \frac{1}{4}x^4 + 2x^6 + C$

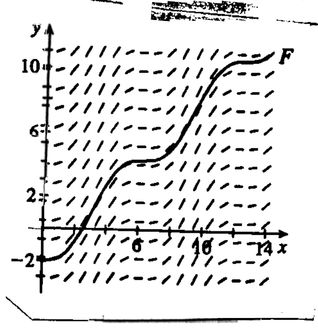
8. $f(x) = \dfrac{3}{x^2} - \dfrac{5}{x^4} = 3x^{-2} - 5x^{-4}$ has domain $(-\infty, 0) \cup (0, \infty)$, so

$$F(x) = \begin{cases} \dfrac{3x^{-1}}{-1} - \dfrac{5x^{-3}}{-3} + C_1 = -\dfrac{3}{x} + \dfrac{5}{3x^3} + C_1 & \text{if } x < 0 \\ -\dfrac{3}{x} + \dfrac{5}{3x^3} + C_2 & \text{if } x > 0 \end{cases}$$

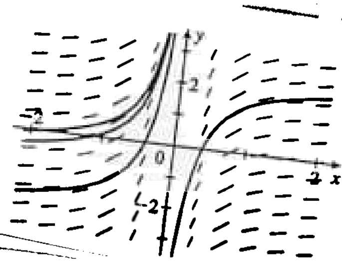
18. $f''(x) = \cos x \quad \Rightarrow \quad f'(x) = \sin x + C \quad \Rightarrow \quad f(x) = -\cos x + Cx + D$

22. $f''(x) = x + x^{1/2} \quad \Rightarrow \quad f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$. $f'(1) = 2 \quad \Rightarrow \quad \frac{1}{2} + \frac{2}{3} + C = 2 \quad \Rightarrow \quad C = \frac{5}{6}$, so $f'(x) = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + \frac{5}{6} \quad \Rightarrow \quad f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x + D$. $f(1) = 1 \quad \Rightarrow \quad \frac{1}{6} + \frac{4}{15} + \frac{5}{6} + D = 1 \quad \Rightarrow \quad D = -\frac{4}{15}$, so $f(x) = \frac{1}{6}x^3 + \frac{4}{15}x^{5/2} + \frac{5}{6}x - \frac{4}{15}$.

30.



34. (a)



(b) The general antiderivative of $f(x) = x^{-2}$ is

$$F(x) = \begin{cases} -1/x + C_1 & \text{if } x < 0 \\ -1/x + C_2 & \text{if } x > 0 \end{cases}$$

since $f(x)$ is not defined at $x = 0$. The graph of the general antiderivatives of $f(x)$ looks like the graph in part (a), as expected.

36. $a(t) = v'(t) = 5 + 4t - 2t^2 \quad \Rightarrow \quad v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + C$. $v(0) = 3 \quad \Rightarrow \quad C = 3$, so $v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + 3$. $v(t) = s'(t) \quad \Rightarrow \quad s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t + D$. $s(0) = 10 \quad \Rightarrow \quad D = 10$, so the particle's position after t seconds is given by $s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t + 10$.

44. $v'(t) = a(t) = -40$. The initial velocity is 50 mi/h $= \frac{50 \cdot 5280}{3600} = \frac{220}{3}$ ft/s, so $v(t) = -40t + \frac{220}{3}$. The car stops when $v(t) = 0 \quad \Leftrightarrow \quad t = \frac{220}{3 \cdot 40} = \frac{11}{6}$. Since $s(t) = -20t^2 + \frac{220}{3}t$, the distance covered is $s\left(\frac{11}{6}\right) = -20\left(\frac{11}{6}\right)^2 + \frac{220}{3} \cdot \frac{11}{6} = \frac{605}{9} \approx 67.2$ ft.