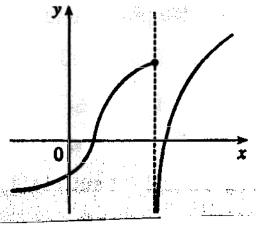
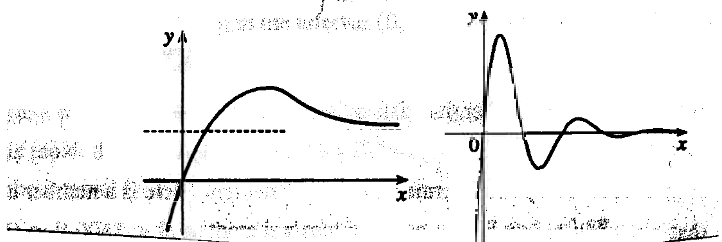
Math 1A Fall 2001: Section 2.5 Solutions

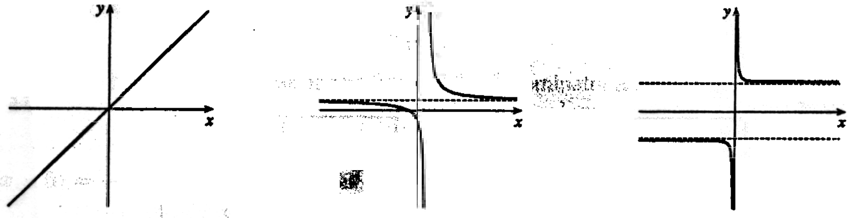
2. (a) The graph of a function can intersect a vertical asymptote in the sense that it can meet but not cross it.



The graph of a function can intersect a horizontal asymptote. It can even intersect its horizontal asymptote an infinite number of times.



(b) The graph of a function can have 0, 1, or 2 horizontal asymptotes. Representative examples are shown.



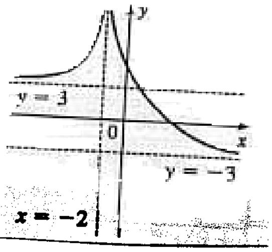
No horizontal asymptote One horizontal asymptote Two horizontal asymptotes

4. (a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to-\infty} g(x) = -2$

(c) $\lim_{x\to3} g(x) = \infty$ (d) $\lim_{x\to0} g(x) = -\infty$

(e) $\lim_{x\to-2^+} g(x) = -\infty$ (f) Vertical: $x = -2$, $x = 0$, $x = 3$; Horizontal: $y = -2$, $y = 2$

8. $\lim_{x\to-2} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = 3$, $\lim_{x\to\infty} f(x) = -3$



18. $$\lim_{x\to\infty}\frac{3x+5}{x-4} = \lim_{x\to\infty}\frac{(3x+5)/x}{(x-4)/x} = \lim_{x\to\infty}\frac{3+5/x}{1-4/x} = \frac{\lim_{x\to\infty} 3 + 5\lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty} 1 - 4\lim_{x\to\infty}\frac{1}{x}} = \frac{3+5(0)}{1-4(0)} = 3$$

22. $$\lim_{x\to\infty}\frac{x+2}{\sqrt{9x^2+1}} = \lim_{x\to\infty}\frac{(x+2)/x}{\sqrt{9x^2+1}/\sqrt{x^2}} = \lim_{x\to\infty}\frac{1+2/x}{\sqrt{9+1/x^2}} = \frac{1+0}{\sqrt{9+0}} = \frac{1}{3}$$

26. If $z = x^4 - x^2$, then $\lim_{x\to\infty} z = \infty$. Thus, $\lim_{x\to\infty}\tan^{-1}(x^4 - x^2) = \lim_{z\to\infty}\tan^{-1} z = \frac{\pi}{2}$.

36. Since the function has vertical asymptotes $x = 1$ and $x = 3$, the denominator of the rational function we are looking for must have factors $(x-1)$ and $(x-3)$. Because the horizontal asymptote is $y = 1$, the degree of the numerator must equal the degree of the denominator, and the ratio of the leading coefficients must be 1. One possibility is

$$f(x) = \frac{x^2}{(x-1)(x-3)}.$$

42. $\lim_{v\to c^-} m = \lim_{v\to c^-}\frac{m_0}{\sqrt{1-v^2/c^2}}$. As $v\to c^-$, $\sqrt{1-v^2/c^2}\to 0^+$, and $m\to\infty$.