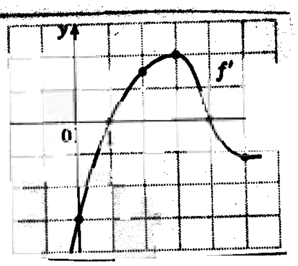
Math 1A Fall 2001: Section 2.8 Solutions

2. From the graph of f, it appears that

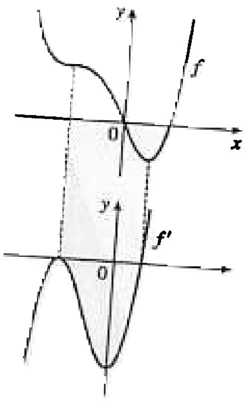
(a) $f'(0) \approx -3$ (b) $f'(1) \approx 0$

(c) $f'(2) \approx 1.5$ (d) $f'(3) \approx 2$

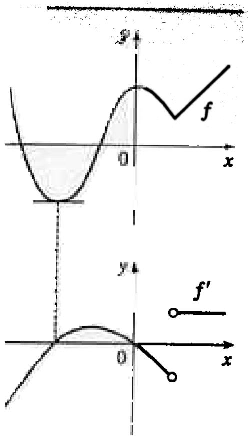
(e) $f'(4) \approx 0$ (f) $f'(5) \approx -1.2$



4.



10.



14. See Figure 1 in Section 3.4.

20.
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left[5-4(x+h)+3(x+h)^2\right]-\left[5-4x+3x^2\right]}{h}$$
$$= \lim_{h\to 0} \frac{\left[5-4x-4h+3x^2+6xh+3h^2\right]-\left[5-4x+3x^2\right]}{h}$$
$$= \lim_{h\to 0} \frac{-4h+6xh+3h^2}{h} = \lim_{h\to 0}(-4+6x+3h) = -4+6x$$

Domain of f = domain of $f' = \mathbb{R}$.

24.
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{3+(x+h)}{1-3(x+h)} - \dfrac{3+x}{1-3x}}{h}$$
$$= \lim_{h\to 0} \frac{(3+x+h)(1-3x)-(3+x)(1-3x-3h)}{h(1-3x-3h)(1-3x)}$$
$$= \lim_{h\to 0} \frac{(3-9x+x-3x^2+h-3hx)-(3-9x-9h+x-3x^2-3hx)}{h(1-3x-3h)(1-3x)}$$
$$= \lim_{h\to 0} \frac{10h}{h(1-3x-3h)(1-3x)} = \lim_{h\to 0} \frac{10}{(1-3x-3h)(1-3x)} = \frac{10}{(1-3x)^2}$$

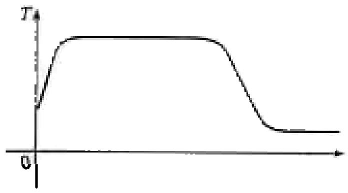
Domain of f = domain of $f' = \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$.

32. (a) g is discontinuous at $x = -2$ (a removable discontinuity), at $x = 0$ (g is not defined there), and at $x = 5$ (a jump discontinuity).

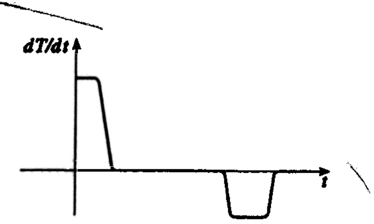
(b) g is not differentiable at the above points (by Theorem 4), and also at $x = -1$ (corner), at $x = 2$ (vertical tangent), and at $x = 4$ (vertical tangent).

36. Where d has horizontal tangents, only c is 0, so $d' = c$. c has negative tangents for $x < 0$ and b is the only graph that is negative for $x < 0$, so $c' = b$. b has positive tangents on $\mathbb{R}$ (except at $x = 0$), and the only graph that is positive on the same domain is a, so $b' = a$. We conclude that $d = f$, $c = f'$, $b = f''$, and $a = f'''$.

48. (a)



(c)



(b) The initial temperature of the water is close to room temperature because of the water that was in the pipes. When the water from the hot water tank starts coming out, dT/dt is large and positive as T increases to the temperature of the water in the tank. In the next phase, $dT/dt = 0$ as the water comes out at a constant, high temperature. After some time, dT/dt becomes small and negative as the contents of the hot water tank are exhausted. Finally, when the hot water has run out, dT/dt is once again 0 as the water maintains its (cold) temperature.