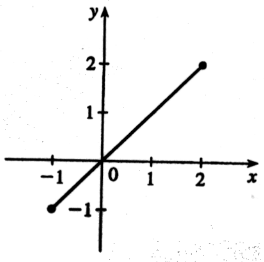
Math 1A Fall 2001: Section 4.2 Solutions

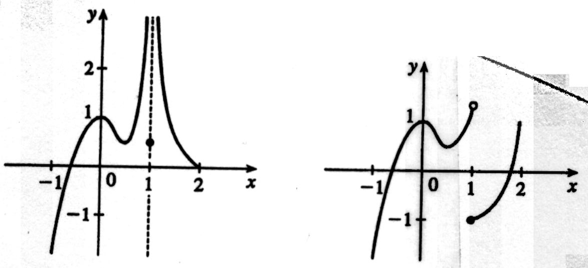
4. Absolute maximum at e; absolute minimum at t; local maxima at c, e, and s; local minima at b, c, d, and r; neither a maximum nor a minimum at a.

6. Absolute maximum value is $f(7) = 5$; absolute minimum value is $f(1) = 0$; local maximum values are $f(0) = 2$, $f(3) = 4$, and $f(5) = 3$; local minimum values are $f(1) = 0$, $f(4) = 2$, and $f(6) = 1$.

12. (a) Note that a local maximum cannot occur at an endpoint.



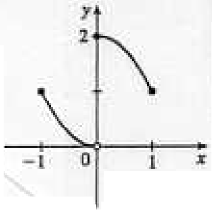
(b)



Note: By the Extreme Value Theorem, f must *not* be continuous.

22. $f(x) = \begin{cases} x^2 & \text{if } -1 \le x < 0 \\ 2 - x^2 & \text{if } 0 \le x \le 1 \end{cases}$

Absolute and local maximum $f(0) = 2$. No absolute or local minimum.



24. $f(x) = x^3 + x^2 - x \;\Rightarrow\; f'(x) = 3x^2 + 2x - 1$. $f'(x) = 0 \;\Rightarrow\; (x+1)(3x-1) = 0 \;\Rightarrow\; x = -1, \frac{1}{3}$. These are the only critical numbers.

34. $f(x) = xe^{2x} \;\Rightarrow\; f'(x) = x(2e^{2x}) + e^{2x} = e^{2x}(2x+1)$. Since e^{2x} is never 0, we have $f'(x) = 0$ only when $2x + 1 = 0 \;\Leftrightarrow\; x = -\frac{1}{2}$. So $-\frac{1}{2}$ is the only critical number.

38. $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$. $f'(x) = -x/\sqrt{9 - x^2} = 0 \;\Leftrightarrow\; x = 0$. $f'(x)$ does not exist at $x = \pm 3$, but neither value is in $[-1, 2]$. $f(-1) = 2\sqrt{2} \approx 2.8$, $f(0) = 3$, $f(2) = \sqrt{5} \approx 2.2$. So $f(0) = 3$ is the absolute maximum and $f(2) = \sqrt{5}$ is the absolute minimum.

44. $f(x) = \dfrac{\ln x}{x}$, $[1, 3]$. $f'(x) = \dfrac{x(1/x) - \ln x}{x^2} = \dfrac{1 - \ln x}{x^2} = 0 \;\Leftrightarrow\; 1 - \ln x = 0 \;\Leftrightarrow\; \ln x = 1 \;\Leftrightarrow\; x = e$. $f(1) = 0/1 = 0$, $f(e) = 1/e \approx 0.368$, $f(3) = (\ln 3)/3 \approx 0.366$. So $f(e) = 1/e$ is the absolute maximum and $f(1) = 0$ is the absolute minimum.