Math 1A Fall 2001: Section 5.4 Solutions

4. (a) $g(-3) = \int_{-3}^{-3} f(t)\,dt = 0$, $g(3) = \int_{-3}^{3} f(t)\,dt = \int_{-3}^{0} f(t)\,dt + \int_{0}^{3} f(t)\,dt = 0$ by symmetry, since the area above the x-axis is the same as the area below the axis.

(b) From the graph, it appears that to the nearest $\frac{1}{2}$,

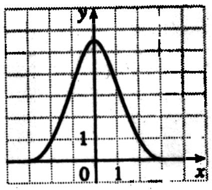
$g(-2) = \int_{-3}^{-2} f(t)\,dt \approx 1$, $g(-1) = \int_{-3}^{-1} f(t)\,dt \approx 3\frac{1}{2}$, and

$g(0) = \int_{-3}^{0} f(t)\,dt \approx 5\frac{1}{2}$.

(c) g is increasing on $(-3, 0)$ because as x increases from -3 to 0, we keep adding more area.

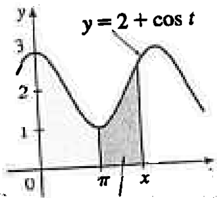
(d) g has a maximum value when we start subtracting area; that is, at $x = 0$.

(e)



(f) The graph of $g'(x)$ is the same as that of $f(x)$, as indicated by FTC1.

6.



$g(x)$ for $x > \pi$

(a) By FTC1, $g(x) = \int_{\pi}^{x} (2 + \cos t)dt \Rightarrow$

$g'(x) = f(x) = 2 + \cos x$.

(b) By FTC2, $g(x) = \int_{\pi}^{x} (2 + \cos t)dt = [2t + \sin t]_{\pi}^{x}$

$= (2x + \sin x) - (2\pi + 0) = 2x + \sin x - 2\pi \Rightarrow$

$g'(x) = 2 + \cos x$.

8. $f(t) = \ln t$ and $g(x) = \int_{1}^{x} \ln t\,dt$, so by FTC1, $g'(x) = f(x) = \ln x$.

12. Let $u = x^2$. Then $\dfrac{du}{dx} = 2x$. Also, $\dfrac{dh}{dx} = \dfrac{dh}{du}\dfrac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_{0}^{x^2} \sqrt{1+r^3}\,dr = \frac{d}{du}\int_{0}^{u} \sqrt{1+r^3}\,dr \cdot \frac{du}{dx} = \sqrt{1+u^3}(2x) = 2x\sqrt{1+(x^2)^3} = 2x\sqrt{1+x^6}.$$

18. For the curve to be concave upward, we must have $y'' > 0$. $y = \displaystyle\int_{0}^{x} \frac{1}{1+t+t^2}\,dt \Rightarrow y' = \frac{1}{1+x+x^2} \Rightarrow$

$y'' = \dfrac{-(1+2x)}{(1+x+x^2)^2}$. For this expression to be positive, we must have $(1+2x) < 0$, since $(1+x+x^2)^2 > 0$ for all x. $(1+2x) < 0 \Leftrightarrow x < -\frac{1}{2}$. Thus, the curve is concave upward on $(-\infty, -\frac{1}{2})$.

20. (a) By FTC1, $g'(x) = f(x)$. So $g'(x) = f(x) = 0$ at $x = 2, 4, 6, 8$, and 10. g has local maxima at $x = 2$ and 6 (since $f = g'$ changes from positive to negative there) and local minima at $x = 4$ and 8. There is no local maximum or minimum at $x = 10$, since f is not defined for $x > 10$.

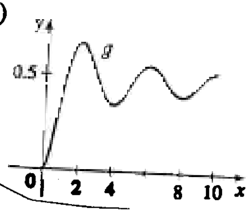
(b) We can see from the graph that $\left|\int_0^2 f\,dt\right| > \left|\int_2^4 f\,dt\right| > \left|\int_4^6 f\,dt\right| > \left|\int_6^8 f\,dt\right| > \left|\int_8^{10} f\,dt\right|$.

So $g(2) = \left|\int_0^2 f\,dt\right|$, $g(6) = \int_0^6 f\,dt = g(2) - \left|\int_2^4 f\,dt\right| + \left|\int_4^6 f\,dt\right|$, and

$g(10) = \int_0^{10} f\,dt = g(6) - \left|\int_6^8 f\,dt\right| + \left|\int_8^{10} f\,dt\right|$. Thus, $g(2) > g(6) > g(10)$, and so the absolute maximum of $g(x)$ occurs at $x = 2$.

(c) g is concave downward on those intervals where $g'' < 0$. But $g'(x) = f(x)$, so $g''(x) = f'(x)$, which is negative on $(1, 3)$, $(5, 7)$ and $(9, 10)$. So g is concave downward on these intervals.

(d)



24. (a) If $x < 0$, then $g(x) = \int_0^x f(t)\,dt = \int_0^x 0\,dt = 0$.

If $0 \le x \le 1$, then $g(x) = \int_0^x f(t)\,dt = \int_0^x t\,dt = \left[\frac{1}{2}t^2\right]_0^x = \frac{1}{2}x^2$.

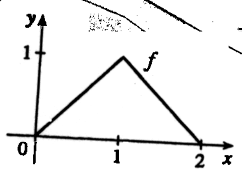
If $1 < x \le 2$, then

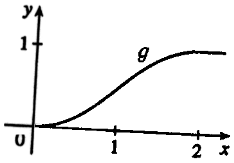
$$\begin{aligned} g(x) &= \int_0^x f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^x f(t)\,dt \\ &= g(1) + \int_1^x (2-t)\,dt = \tfrac{1}{2}(1)^2 + \left[2t - \tfrac{1}{2}t^2\right]_1^x \\ &= \tfrac{1}{2} + \left(2x - \tfrac{1}{2}x^2\right) - \left(2 - \tfrac{1}{2}\right) = 2x - \tfrac{1}{2}x^2 - 1. \end{aligned}$$

If $x > 2$, then $g(x) = \int_0^x f(t)\,dt = g(2) + \int_2^x 0\,dt = 1 + 0 = 1$. So

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } 0 \le x \le 1 \\ 2x - \frac{1}{2}x^2 - 1 & \text{if } 1 < x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$$

(b)





(c) f is not differentiable at its corners at $x = 0$, 1, and 2. f is differentiable on $(-\infty, 0)$

g is differentiable on (