Math 1A Fall 2001: Section 5.5 Solutions

2. Let $u = 4 + x^2$. Then $du = 2x\,dx$, so $\int x(4+x^2)^{10}\,dx = \int u^{10}\left(\frac{1}{2}\,du\right) = \frac{1}{2}\cdot\frac{1}{11}u^{11} + C = \frac{1}{22}(4+x^2)^{11} + C$.

6. Let $u = \sin\theta$. Then $du = \cos\theta\,d\theta$, so $\int e^{\sin\theta}\cos\theta\,d\theta = \int e^u\,du = e^u + C = e^{\sin\theta} + C$.

12. Let $u = 2 - x$. Then $du = -dx$, so $\int (2-x)^6\,dx = \int u^6(-du) = -\frac{1}{7}u^7 + C = -\frac{1}{7}(2-x)^7 + C$.

18. Let $u = 3 - 5y$. Then $du = -5\,dy$, so

$$\int \sqrt[5]{3-5y}\,dy = \int u^{1/5}\left(-\tfrac{1}{5}\,du\right) = -\tfrac{1}{5}\cdot\tfrac{5}{6}u^{6/5} + C = -\tfrac{1}{6}(3-5y)^{6/5} + C$$

28. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$.

32. Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{1+x^4}\,dx = \int \frac{\frac{1}{2}\,du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}(x^2) + C$.

44. Let $u = 3x + 1$, so $du = 3dx$. When $x = 1$, $u = 4$; when $x = 2$, $u = 7$. Thus,

$$\int_1^2 \frac{dx}{3x+1} = \int_4^7 \frac{1}{u}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\left[\ln|u|\right]_4^7 = \frac{1}{3}(\ln 7 - \ln 4) = \frac{1}{3}\ln\frac{7}{4}$$

48. Let $u = 1 + 2x$, so $x = \frac{1}{2}(u-1)$ and $du = 2\,dx$. When $x = 0$, $u = 1$; when $x = 4$, $u = 9$. Thus,

$$\int_0^4 \frac{x\,dx}{\sqrt{1+2x}} = \int_1^9 \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\frac{du}{2} = \frac{1}{4}\int_1^9\left(u^{1/2} - u^{-1/2}\right)du = \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9$$

$$= \frac{1}{4}\cdot\frac{2}{3}\left[u^{3/2} - 3u^{1/2}\right]_1^9 = \frac{1}{6}\left[(27-9) - (1-3)\right] = \frac{20}{6} = \frac{10}{3}$$

56. Let $u = x^2$. Then $du = 2x\,dx$ and the limits are unchanged ($0^2 = 0$ and $1^2 = 1$), so $I = \int_0^1 x\sqrt{1-x^4}\,dx = \frac{1}{2}\int_0^1\sqrt{1-u^2}\,du$. But this integral can be interpreted as the area of a quarter-circle with radius 1. So $I = \frac{1}{2}\cdot\frac{1}{4}(\pi\cdot 1^2) = \frac{1}{8}\pi$.