2.7

2. As h decreases, the line PQ becomes steeper, so its slope increases. So $0 < \frac{f(4)-f(2)}{4-2} < \frac{f(3)-f(2)}{3-2} < \lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$. Thus, $0 < \frac{1}{2}[f(4)-f(2)] < f(3)-f(2) < f'(2)$.

3. $g'(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$. Thus, $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$.

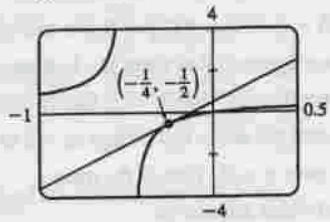
4. Since $(4,3)$ is on $y=f(x)$, $f(4)=3$. The slope of the tangent line between $(0,2)$ and $(4,3)$ is $\frac{1}{4}$, so $f'(4)=\frac{1}{4}$.

10. (a) $G'(a) = \lim_{h\to 0}\frac{G(a+h)-G(a)}{h} = \lim_{h\to 0}\frac{\frac{a+h}{1+2(a+h)}-\frac{a}{1+2a}}{h}$

$= \lim_{h\to 0}\frac{a+2a^2+h+2ah-a-2a^2-2ah}{h(1+2a+2h)(1+2a)} = \lim_{h\to 0}\frac{1}{(1+2a+2h)(1+2a)} = (1+2a)^{-2}$

So the slope of the tangent at the point $\left(-\frac{1}{4},-\frac{1}{2}\right)$ is $m=\left[1+2\left(-\frac{1}{4}\right)\right]^{-2}=4$, and thus an equation is $y+\frac{1}{2}=4\left(x+\frac{1}{4}\right)$ or $y=4x+\frac{1}{2}$.

(b)



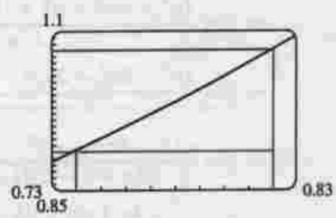
12. (a) $g'\left(\frac{\pi}{4}\right) = \lim_{h\to 0}\frac{g\left(\frac{\pi}{4}+h\right)-g\left(\frac{\pi}{4}\right)}{h} = \lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\left(\frac{\pi}{4}\right)}{h}$.

So let $G(h)=\frac{\tan\left(\frac{\pi}{4}+h\right)-1}{h}$. We calculate:

h	$G(h)$
0.1	2.2305
0.01	2.0203
0.001	2.0020
0.0001	2.0002
−0.1	1.8237
−0.01	1.9803
−0.001	1.9980
−0.0001	1.9998

We estimate that $g'\left(\frac{\pi}{4}\right)=2$.

(b)



From the graph, we estimate that the slope of the tangent is about $\frac{1.07-0.91}{0.82-0.74}=\frac{0.16}{0.08}=2$.

28. (a) $f'(5)$ is the rate of growth of the bacteria population when $t=5$ hours. Its units are bacteria per hour.

(b) With unlimited space and nutrients, f' should increase as t increases; so $f'(5) < f'(10)$. If the supply of nutrients is limited, the growth rate slows down at some point in time, and the opposite may be true.

33. $C'(1980)$ is the rate of change of U.S. cash per capita in circulation with respect to time. To estimate the value of $C'(1980)$, we will average the difference quotients obtained using the years 1970 and 1990.

Let $A=\frac{C(1970)-C(1980)}{1970-1980}=\frac{265-571}{-10}=30.6$ and $B=\frac{C(1990)-C(1980)}{1990-1980}=\frac{1063-571}{10}=49.2$.

Then $C'(1980)=\lim_{t\to 1980}\frac{C(t)-C(1980)}{t-1980}\approx\frac{A+B}{2}=39.9$ dollars per year.

36. Since $f(x) = x^2 \sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$f'(0) = \lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \lim\limits_{h \to 0} \dfrac{h^2 \sin(1/h) - 0}{h} = \lim\limits_{h \to 0} h \sin(1/h)$. Since $-1 \leq \sin \dfrac{1}{h} \leq 1$, we have $-|h| \leq |h| \sin \dfrac{1}{h} \leq |h| \quad \Rightarrow \quad -|h| \leq h \sin \dfrac{1}{h} \leq |h|$. Because $\lim\limits_{h \to 0} (-|h|) = 0$ and $\lim\limits_{h \to 0} |h| = 0$, we know that $\lim\limits_{h \to 0} \left(h \sin \dfrac{1}{h} \right) = 0$ by the Squeeze Theorem. Thus, $f'(0) = 0$.