5.3 Evaluating Definite Integrals

1. If $w'(t)$ is the rate of change of weight in pounds per year, then $w(t)$ represents the weight in pounds of the child at age t. We know from the Total Change Theorem that $\int_5^{10} w'(t)\,dt = w(10) - w(5)$, so the integral represents the increase in the child's weight (in pounds) between the ages of 5 and 10.

2. $\int_a^b I(t)\,dt = \int_a^b Q'(t)\,dt = Q(b) - Q(a)$ by the Total Change Theorem, so it represents the change in the charge Q from time $t = a$ to $t = b$.

4. By the Total Change Theorem, $\int_0^{15} n'(t)\,dt = n(15) - n(0) = n(15) - 100$ represents the increase in the bee population in 15 weeks. So $100 + \int_0^{15} n'(t)\,dt = n(15)$ represents the total bee population after 15 weeks.

12. $\int_0^4 (1 + 3y - y^2)\,dy = \left[y + \frac{3}{2}y^2 - \frac{1}{3}y^3\right]_0^4 = \left(4 + \frac{3}{2} \cdot 16 - \frac{1}{3} \cdot 64\right) - (0) = \frac{20}{3}$

18. $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}\,dx = \int_1^4 x^{-1/2}\,dx = \left[\frac{x^{1/2}}{1/2}\right]_1^4 = \left[2x^{1/2}\right]_1^4 = 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = 2$

28. $\int_{\ln 3}^{\ln 6} 8e^x\,dx = [8e^x]_{\ln 3}^{\ln 6} = 8\left(e^{\ln 6} - e^{\ln 3}\right) = 8(6 - 3) = 24$

49. $A = \int_0^2 (2y - y^2)\,dy = \left[y^2 - \frac{1}{3}y^3\right]_0^2 = \left(4 - \frac{8}{3}\right) - 0 = \frac{4}{3}$

57. Let s be the position of the car. We know from Equation 2 that $s(100) - s(0) = \int_0^{100} v(t)\,dt$. We use the Midpoint Rule for $0 \le t \le 100$ with $n = 5$. Note that the length of each of the five time intervals is 20 seconds $= \frac{20}{3600}$ hour $= \frac{1}{180}$ hour. So the distance traveled is

$$\begin{aligned}\int_0^{100} v(t)\,dt &\approx \tfrac{1}{180}[v(10) + v(30) + v(50) + v(70) + v(90)]\\ &= \tfrac{1}{180}(38 + 58 + 51 + 53 + 47)\\ &= \tfrac{247}{180} \approx 1.4 \text{ miles}\end{aligned}$$