5.5

4. Let $u = \sqrt{x}$. Then $du = \dfrac{1}{2\sqrt{x}}\,dx$, so $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u\,(2\,du) = 2(-\cos u) + C = -2\cos\sqrt{x} + C.$

5. Let $u = 1 + 2x$. Then $du = 2\,dx$, so

$$\int \frac{4}{(1+2x)^3}\,dx = 4\int u^{-3}\left(\tfrac{1}{2}\,du\right) = 2\frac{u^{-2}}{-2} + C = -\frac{1}{u^2} + C = -\frac{1}{(1+2x)^2} + C$$

16. Let $u = 1 - t^3$. Then $du = -3t^2\,dt$, so

$$\int t^2\cos\left(1-t^3\right)dt = \int \cos u\left(-\tfrac{1}{3}\,du\right) = -\tfrac{1}{3}\sin u + C = -\tfrac{1}{3}\sin\left(1-t^3\right) + C$$

18. Let $u = 3 - 5y$. Then $du = -5\,dy$, so

$$\int \sqrt[5]{3-5y}\,dy = \int u^{1/5}\left(-\tfrac{1}{5}\,du\right) = -\tfrac{1}{5}\cdot\tfrac{5}{6}u^{6/5} + C = -\tfrac{1}{6}(3-5y)^{6/5} + C$$

28. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\displaystyle\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C.$

45. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 \left(u^{3/2} + u^{1/2}\right)du = \left[\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\right]_0^1 = \tfrac{2}{5} + \tfrac{2}{3} = \tfrac{16}{15}$$

51. Let $u = \ln x$, so $du = \dfrac{dx}{x}$. When $x = e$, $u = 1$; when $x = e^4$; $u = 4$. Thus,

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 u^{-1/2}\,du = 2\left[u^{1/2}\right]_1^4 = 2(2-1) = 2$$

61. Let $u = 2x$. Then $du = 2\,dx$, so $\int_0^2 f(2x)\,dx = \int_0^4 f(u)\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\int_0^4 f(u)\,du = \tfrac{1}{2}(10) = 5.$