# Worksheet on Logarithmic Differentiation (Solutions)

Math 1a: Introduction to Calculus

21 March 2005

*For each of the following, differentiate the function first using any rule you want, then using logarithmic differentiation:*

**1**. $y = x^2$

*Solution.* If $y = x^2$, then

$$\ln y = \ln(x^2) = 2\ln x.$$

Differentiating,

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x},$$

so

$$\frac{dy}{dx} = \frac{2y}{x} = \frac{2x^2}{x} = 2x.$$

This is the same answer that we could have gotten with the power rule. □

**2**. $y = e^x$

*Solution.*

$$\begin{aligned} y = e^x &\implies \ln y = x \\ \frac{1}{y}\frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= y = e^x. \end{aligned}$$

□

**3**. $y = \sqrt{x^2+1}$

*Solution.*

$$\begin{aligned} y = \sqrt{x^2+1} &\implies \ln y = \frac{1}{2}\ln(x^2+1) \\ \frac{1}{y}\frac{dy}{dx} &= \frac{1}{2}\frac{2x}{x^2+1} \\ \frac{dy}{dx} &= \frac{xy}{x^2+1} = \frac{x\sqrt{x^2+1}}{x^2+1} = \frac{x}{\sqrt{x^2+1}}. \end{aligned}$$

Again, notice this is the same answer we could have gotten without logarithmic differentiation. □

**4**. $y = x\sin x$

*Solution.*

$$\begin{aligned} \ln y &= \ln x + \ln(\sin x) \\ \frac{1}{y}\frac{dy}{dx} &= \frac{1}{x} + \frac{\cos x}{\sin x} \\ \frac{dy}{dx} &= \left(\frac{1}{x} + \frac{\cos x}{\sin x}\right)(x\sin x) \\ &= \sin x + x\cos x. \end{aligned}$$

□

**5**. $y = \frac{x}{x^2+2}$

*Solution.*

$$\begin{aligned}
\ln y &= \ln x - \ln(x^2+2) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{1}{x} - \frac{2x}{x^2+2} \\
\frac{dy}{dx} &= \left(\frac{1}{x} - \frac{2x}{x^2+2}\right)\frac{x}{x^2+2} \\
&= \frac{(x^2+2) - x(2x)}{(x^2+2)^2}.
\end{aligned}$$

□

**6**. $y = \sqrt{(x^2+1)(x-1)^2}$.

*Solution.*

$$\begin{aligned}
\ln y &= \frac{1}{2}\left(\ln(x^2+1) + 2\ln(x-1)\right) \\
\frac{1}{y}\frac{dy}{dx} &= \frac{x}{x^2+1} + \frac{1}{x-1} \\
\frac{dy}{dx} &= \left(\frac{x}{x^2+1} + \frac{1}{x-1}\right)\sqrt{(x^2+1)(x-1)^2}.
\end{aligned}$$

□

Use logarithmic differentiation to find the following derivatives:

**7**. $y = (x+1)^x$

*Solution.*

$$\begin{aligned}
\ln y &= x\ln(x+1) \\
\frac{1}{y}\frac{dy}{dx} &= \ln(x+1) + \frac{x}{x+1} \\
\frac{dy}{dx} &= \left(\ln(x+1) + \frac{x}{x+1}\right)(x+1)^x.
\end{aligned}$$

□

**8**. $y = x^{x+1}$

*Solution.*

$$\begin{aligned}
\ln y &= (x+1)\ln x \\
\frac{1}{y}\frac{dy}{dx} &= \ln x + \frac{x+1}{x} \\
\frac{dy}{dx} &= \left(\ln x + \frac{x+1}{x}\right)x^{x+1}.
\end{aligned}$$

□

**9**. $y = (\sqrt{x})^x$

*Solution.*

$$\begin{aligned}
\ln y &= x\ln\sqrt{x} = \tfrac{1}{2}x\ln x \\
\frac{1}{y}\frac{dy}{dx} &= \tfrac{1}{2}(\ln x + 1) \\
\frac{dy}{dx} &= \tfrac{1}{2}(\ln x + 1)(\sqrt{x})^x.
\end{aligned}$$

□

**10**. $y = x^{\sqrt{x}}$

*Solution.*

$$\begin{aligned}
\ln y &= \sqrt{x}\ln x \\
\frac{1}{y}\frac{dy}{dx} &= \frac{1}{2\sqrt{x}}\ln x + \frac{\sqrt{x}}{x} \\
\frac{dy}{dx} &= \frac{(\ln x + 2)x^{\sqrt{x}}}{2\sqrt{x}}.
\end{aligned}$$

□