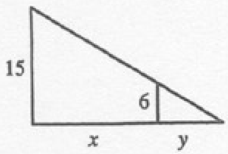
Math 1a Homework Solutions
Section 4.1

4. $y = \sqrt{1+x^3} \Rightarrow \dfrac{dy}{dt} = \dfrac{dy}{dx}\dfrac{dx}{dt} = \frac{1}{2}(1+x^3)^{-1/2}\,(3x^2)\,\dfrac{dx}{dt} = \dfrac{3x^2}{2\sqrt{1+x^3}}\,\dfrac{dx}{dt}$. With $\dfrac{dy}{dt} = 4$ when $x = 2$ and $y = 3$, we have $4 = \dfrac{3(4)}{2(3)}\,\dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = 2$ cm/s.

8. (a) Given: a man 6 ft tall walks away from a street light mounted on a 15-ft-tall pole at a rate of 5 ft/s. If we let t be time (in s) and x be the distance from the pole to the man (in ft), then we are given that $dx/dt = 5$ ft/s.

(b) Unknown: the rate at which the tip of his shadow is moving when he is 40 ft from the pole. If we let y be the distance from the man to the tip of his shadow (in ft), then we want to find $\dfrac{d}{dt}(x+y)$ when $x = 40$ ft.

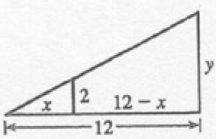
(c)



(d) By similar triangles, $\dfrac{15}{6} = \dfrac{x+y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x \Rightarrow y = \frac{2}{3}x$.

(e) The tip of the shadow moves at a rate of $\dfrac{d}{dt}(x+y) = \dfrac{d}{dt}\left(x + \frac{2}{3}x\right) = \dfrac{5}{3}\dfrac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/s.

10.



We are given that $\dfrac{dx}{dt} = 1.6$ m/s. By similar triangles, $\dfrac{y}{12} = \dfrac{2}{x} \Rightarrow y = \dfrac{24}{x}$
$\Rightarrow \dfrac{dy}{dt} = -\dfrac{24}{x^2}\dfrac{dx}{dt} = -\dfrac{24}{x^2}(1.6)$. When $x = 8$, $\dfrac{dy}{dt} = -\dfrac{24(1.6)}{64} = -0.6$ m/s, so the shadow is decreasing at a rate of 0.6 m/s.

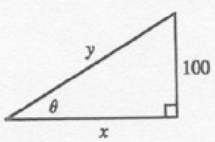
16. Let D denote the distance from the origin $(0,0)$ to the point on the curve $y = \sqrt{x}$.

$D = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (\sqrt{x})^2} = \sqrt{x^2 + x} \Rightarrow$

$\dfrac{dD}{dt} = \frac{1}{2}\left(x^2 + x\right)^{-1/2}(2x+1)\,\dfrac{dx}{dt} = \dfrac{2x+1}{2\sqrt{x^2+x}}\,\dfrac{dx}{dt}$. With $\dfrac{dx}{dt} = 3$ when $x = 4$,

$\dfrac{dD}{dt} = \dfrac{9}{2\sqrt{20}}(3) = \dfrac{27}{4\sqrt{5}} \approx 3.02$ cm/s.

22.

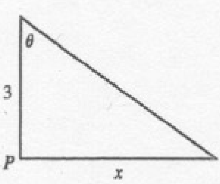


We are given $dx/dt = 8$ ft/s. $\cot\theta = \dfrac{x}{100} \Rightarrow x = 100\cot\theta \Rightarrow$

$\dfrac{dx}{dt} = -100\csc^2\theta\,\dfrac{d\theta}{dt} \Rightarrow \dfrac{d\theta}{dt} = -\dfrac{\sin^2\theta}{100}\cdot 8$. When $y = 200$,

$\sin\theta = \dfrac{100}{200} = \dfrac{1}{2} \Rightarrow \dfrac{d\theta}{dt} = -\dfrac{(1/2)^2}{100}\cdot 8 = -\dfrac{1}{50}$ rad/s. The angle is decreasing at a rate of $\frac{1}{50}$ rad/s.

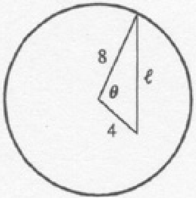
30.



We are given that $\dfrac{d\theta}{dt} = 4(2\pi) = 8\pi$ rad/min. $x = 3\tan\theta \Rightarrow$

$\dfrac{dx}{dt} = 3\sec^2\theta\,\dfrac{d\theta}{dt}$. When $x = 1$, $\tan\theta = \frac{1}{3}$, so $\sec^2\theta = 1 + \left(\frac{1}{3}\right)^2 = \frac{10}{9}$ and

$\dfrac{dx}{dt} = 3\left(\frac{10}{9}\right)(8\pi) = \frac{80\pi}{3} \approx 83.8$ km/min.

34. 

The hour hand of a clock goes around once every 12 hours or, in radians per hour, $\frac{2\pi}{12} = \frac{\pi}{6}$ rad/h. The minute hand goes around once an hour, or at the rate of 2π rad/h. So the angle θ between them (measuring clockwise from the minute hand to the hour hand) is changing at the rate of $d\theta/dt = \frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$ rad/h. Now, to relate θ to ℓ, we use the Law of Cosines: $\ell^2 = 4^2 + 8^2 - 2 \cdot 4 \cdot 8 \cdot \cos\theta = 80 - 64\cos\theta$ $(\star)$.

Differentiating implicitly with respect to t, we get $2\ell\,\dfrac{d\ell}{dt} = -64(-\sin\theta)\dfrac{d\theta}{dt}$. At 1:00, the angle between the two hands is one-twelfth of the circle, that is, $\frac{2\pi}{12} = \frac{\pi}{6}$ radians. We use $(\star)$ to find ℓ at 1:00:

$\ell = \sqrt{80 - 64\cos\frac{\pi}{6}} = \sqrt{80 - 32\sqrt{3}}$. Substituting, we get $2\ell\,\dfrac{d\ell}{dt} = 64\sin\frac{\pi}{6}\left(-\frac{11\pi}{6}\right) \Rightarrow$

$\dfrac{d\ell}{dt} = \dfrac{64\left(\frac{1}{2}\right)\left(-\frac{11\pi}{6}\right)}{2\sqrt{80 - 32\sqrt{3}}} = -\dfrac{88\pi}{3\sqrt{80 - 32\sqrt{3}}} \approx -18.6$. So at 1:00, the distance between the tips of the hands is decreasing at a rate of 18.6 mm/h ≈ 0.005 mm/s.