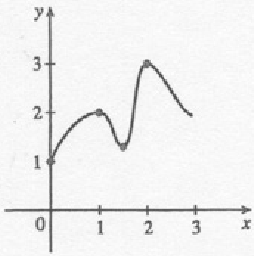
Math 1a Homework Solutions
Section 4.2

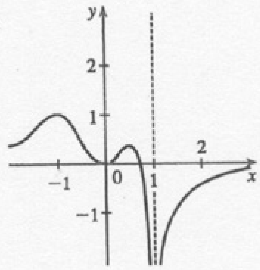
4. Absolute maximum at e; absolute minimum at t; local maxima at c, e, and s; local minima at b, c, d, and r; neither a maximum nor a minimum at a.

6. Absolute maximum value is $f(7) = 5$; absolute minimum value is $f(1) = 0$; local maximum values are $f(0) = 2$, $f(3) = 4$, and $f(5) = 3$; local minimum values are $f(1) = 0$, $f(4) = 2$, and $f(6) = 1$.

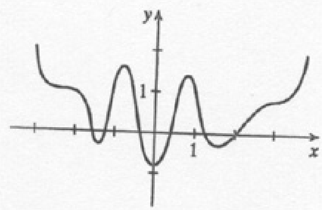
10.



14. (a)



(b)



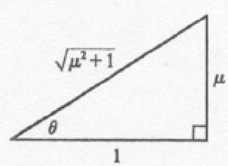
44. $f(x) = \dfrac{\ln x}{x}$, $[1, 3]$. $f'(x) = \dfrac{x(1/x) - \ln x}{x^2} = \dfrac{1 - \ln x}{x^2} = 0 \Leftrightarrow 1 - \ln x = 0 \Leftrightarrow \ln x = 1 \Leftrightarrow x = e$.
$f(1) = 0/1 = 0$, $f(e) = 1/e \approx 0.368$, $f(3) = (\ln 3)/3 \approx 0.366$. So $f(e) = 1/e$ is the absolute maximum and $f(1) = 0$ is the absolute minimum.

52. $F = \dfrac{\mu W}{\mu \sin\theta + \cos\theta} \Rightarrow \dfrac{dF}{d\theta} = \dfrac{(\mu\sin\theta + \cos\theta)(0) - \mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} = \dfrac{-\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}$. So

$\dfrac{dF}{d\theta} = 0 \Rightarrow \mu\cos\theta - \sin\theta = 0 \Rightarrow \mu = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$. Substituting $\tan\theta$ for μ in F gives us

$$F = \frac{(\tan\theta)W}{(\tan\theta)\sin\theta + \cos\theta} = \frac{W\tan\theta}{\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta} = \frac{W\tan\theta\cos\theta}{\sin^2\theta + \cos^2\theta} = \frac{W\sin\theta}{1} = W\sin\theta.$$

If $\tan\theta = \mu$, then $\sin\theta = \dfrac{\mu}{\sqrt{\mu^2+1}}$ (see the figure), so $F = \dfrac{\mu}{\sqrt{\mu^2+1}}W$. We compare this with the value of F at the endpoints: $F(0) = \mu W$ and $F(\frac{\pi}{2}) = W$.



Now because $\dfrac{\mu}{\sqrt{\mu^2+1}} \le 1$ and $\dfrac{\mu}{\sqrt{\mu^2+1}} \le \mu$, we have that $\dfrac{\mu}{\sqrt{\mu^2+1}}W$ is less than or equal to each of $F(0)$ and $F(\frac{\pi}{2})$. Hence, $\dfrac{\mu}{\sqrt{\mu^2+1}}W$ is the absolute minimum value of $F(\theta)$, and it occurs when $\tan\theta = \mu$.