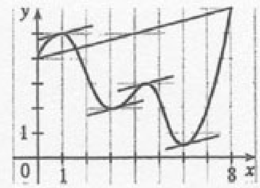
Math 1a Homework Solutions
Section 4.3

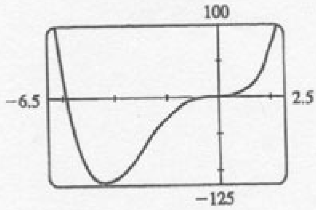
1. $\frac{f(8)-f(0)}{8-0} = \frac{6-4}{8} = \frac{1}{4}$. The values of c which satisfy $f'(c) = \frac{1}{4}$ seem to be about $c = 0.8$, 3.2, 4.4, and 6.1.



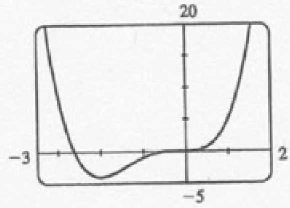
43. $f(x) = \tan x - x \Rightarrow f'(x) = \sec^2 x - 1 > 0$ for $0 < x < \frac{\pi}{2}$ since $\sec^2 x > 1$ for $0 < x < \frac{\pi}{2}$. So f is increasing on $(0, \frac{\pi}{2})$. Thus, $f(x) > f(0) = 0$ for $0 < x < \frac{\pi}{2} \Rightarrow \tan x - x > 0 \Rightarrow \tan x > x$ for $0 < x < \frac{\pi}{2}$.

46. By the Mean Value Theorem, $\frac{f(5)-f(2)}{5-2} = f'(c)$ for some $c \in (2,5)$. Since $1 \le f'(x) \le 4$, we have $1 \le \frac{f(5)-f(2)}{5-2} \le 4 \Leftrightarrow 1 \le \frac{f(5)-f(2)}{3} \le 4 \Leftrightarrow 3 \le f(5) - f(2) \le 12$.

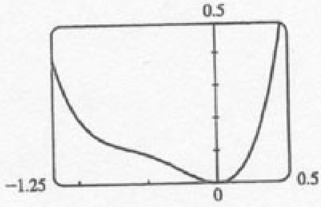
50. $P(x) = x^4 + cx^3 + x^2 \Rightarrow P'(x) = 4x^3 + 3cx^2 + 2x \Rightarrow P''(x) = 12x^2 + 6cx + 2$. The graph of $P''(x)$ is a parabola. If $P''(x)$ has two roots, then it changes sign twice and so has two inflection points. This happens when the discriminant of $P''(x)$ is positive, that is, $(6c)^2 - 4 \cdot 12 \cdot 2 > 0 \Leftrightarrow 36c^2 - 96 > 0 \Leftrightarrow |c| > \frac{2\sqrt{6}}{3} \approx 1.63$. If $36c^2 - 96 = 0 \Leftrightarrow c = \pm\frac{2\sqrt{6}}{3}$, $P''(x)$ is 0 at one point, but there is still no inflection point since $P''(x)$ never changes sign, and if $36c^2 - 96 < 0 \Leftrightarrow |c| < \frac{2\sqrt{6}}{3}$, then $P''(x)$ never changes sign, and so there is no inflection point.



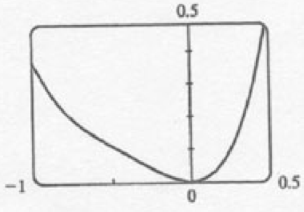
$c = 6$



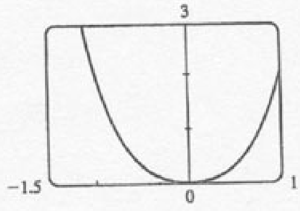
$c = 3$



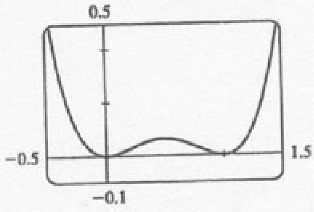
$c = 1.8$



$c = \frac{2\sqrt{6}}{3}$



$c = 0$



$c = -2$

For large positive c, the graph of f has two inflection points and a large dip to the left of the y-axis. As c decreases, the graph of f becomes flatter for $x < 0$, and eventually the dip rises above the x-axis, and then disappears entirely, along with the inflection points. As c continues to decrease, the dip and the inflection points reappear, to the right of the origin.