Math 1A Fall 2001: Section 2.6 Solutions

2. (a) Average velocity $= \dfrac{\Delta s}{\Delta t} = \dfrac{f(a+h)-f(a)}{(a+h)-a} = \dfrac{f(a+h)-f(a)}{h}$

(b) Instantaneous velocity $= \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

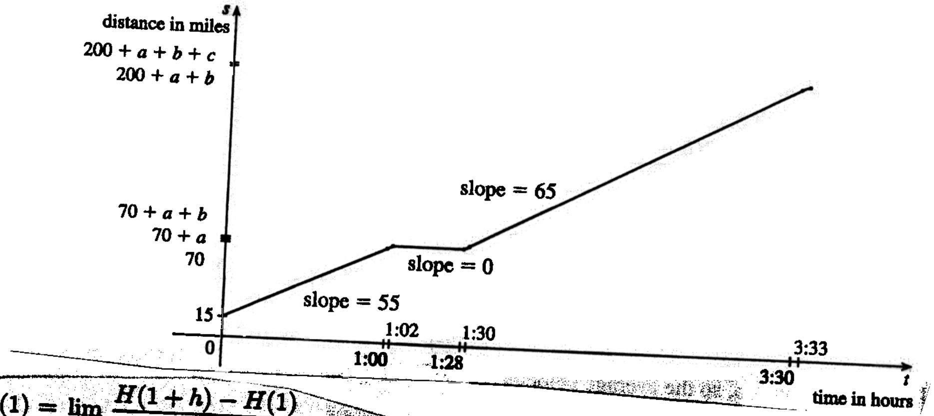
6. (a) (i) $m = \lim\limits_{x\to -1} \dfrac{f(x)-f(-1)}{x-(-1)} = \lim\limits_{x\to -1} \dfrac{x^3-(-1)}{x+1} = \lim\limits_{x\to -1} \dfrac{(x+1)(x^2-x+1)}{x+1}$
$= \lim\limits_{x\to -1} (x^2-x+1) = 3$

(ii) $m = \lim\limits_{h\to 0} \dfrac{f(-1+h)-f(-1)}{h} = \lim\limits_{h\to 0} \dfrac{(-1+h)^3-(-1)}{h} = \lim\limits_{h\to 0} \dfrac{h^3-3h^2+3h-1+1}{h}$
$= \lim\limits_{h\to 0} (h^2-3h+3) = 3$

(b) $y-(-1) = 3[x-(-1)] \Leftrightarrow y+1 = 3x+3 \Leftrightarrow y = 3x+2$

18. Using (1), $m = \lim\limits_{x\to 0} \dfrac{\frac{2x}{(x+1)^2}-0}{x-0} = \lim\limits_{x\to 0} \dfrac{2x}{x(x+1)^2} = \lim\limits_{x\to 0} \dfrac{2}{(x+1)^2} = \dfrac{2}{1^2} = 2.$
Tangent line: $y-0 = 2(x-0) \Leftrightarrow y = 2x$

14. Let a denote the distance traveled from 1:00 to 1:02, b from 1:28 to 1:30, and c from 3:30 to 3:33, where all the times are relative to $t=0$ at the beginning of the trip.



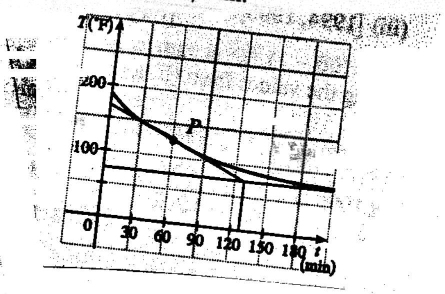
16. (a) $v(1) = \lim\limits_{h\to 0} \dfrac{H(1+h)-H(1)}{h}$
$= \lim\limits_{h\to 0} \dfrac{(58+58h-0.83-1.66h-0.83h^2)-57.17}{h} = \lim\limits_{h\to 0}(56.34-0.83h) = 56.34$ m/s

(b) $v(a) = \lim\limits_{h\to 0} \dfrac{H(a+h)-H(a)}{h}$
$= \lim\limits_{h\to 0} \dfrac{(58a+58h-0.83a^2-1.66ah-0.83h^2)-(58a-0.83a^2)}{h}$
$= \lim\limits_{h\to 0} (58-1.66a-0.83h) = 58-1.66a$ m/s

(c) The arrow strikes the moon when the height is 0, that is, $58t-0.83t^2=0 \Leftrightarrow t(58-0.83t)=0 \Leftrightarrow$ $t = \frac{58}{0.83} \approx 69.9$ s (since t can't be 0).

(d) Using the time from part (c), $v\left(\frac{58}{0.83}\right) = 58-1.66\left(\frac{58}{0.83}\right) = -58$ m/s. Thus, the arrow will have a velocity of -58 m/s.

20. The slope of the tangent (that is, the rate of change of temperature with respect to time) at $t = 1$ h seems to be about $\frac{75-168}{132-0} \approx -0.7\,°\text{F/min}$.



24. (a) (i) $[1996, 1998]$: $\dfrac{N(1998)-N(1996)}{1998-1996} = \dfrac{1886-1015}{2} = \dfrac{871}{2} = 435.5$ locations/year

(ii) $[1996, 1997]$: $\dfrac{N(1997)-N(1996)}{1997-1996} = \dfrac{1412-1015}{1} = 397$ locations/year

(iii) $[1995, 1996]$: $\dfrac{N(1996)-N(1995)}{1996-1995} = \dfrac{1015-676}{1} = 339$ locations/year

(b) Using the values from (ii) and (iii), we have $\dfrac{397+339}{2} = \dfrac{736}{2} = 368$ locations/year.

(c) Estimating A as $(1995, 660)$ and B as $(1997, 1350)$, the slope at 1996 is $\dfrac{1350-660}{1997-1995} = \dfrac{690}{2} = 345$ locations/year.