Math 1A Fall 2001: Section 2.9 Solutions

6. (a) From Exercise 2.8.18(e) for $f(x) = x^3$, $f'(x) = 3x^2$, so $f'(1) = 3$.

(b) $L(x) = f(1) + f'(1)(x-1)$

$= 1 + 3(x-1) = 3x - 2$

The estimates using L are all underestimates of the actual function values.

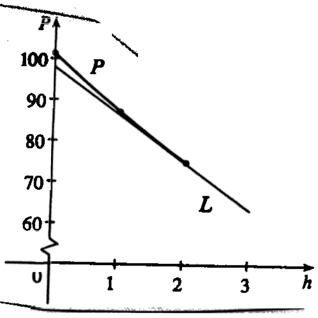
x	$L(x) = 3x - 2$	$f(x) = x^3$
0.9	0.7	0.729
0.95	0.85	0.857375
0.99	0.97	0.970299
1.01	1.03	1.030301
1.05	1.15	1.157625
1.1	1.3	1.331

8. $P'(2) \approx \dfrac{P(1) - P(2)}{1 - 2} = \dfrac{87.1 - 74.9}{-1} = -12.2$ kilopascals/km.

$P(3) \approx P(2) + P'(2)(3-2) \approx 74.9 - 12.2(1) = 62.7$ kPa.

From the figure, we estimate the slope of the tangent line at $h = 2$ to be $\frac{98-63}{0-3} = -\frac{35}{3}$. Then the linear approximation becomes

$P(3) \approx P(2) + P'(2) \cdot 1 \approx 74.9 - \frac{35}{3} \approx 63.23$ kPa.



10. Let $A = \dfrac{N(1980) - N(1985)}{1980 - 1985} = \dfrac{15.0 - 17.0}{-5} = 0.4$ and $B = \dfrac{N(1990) - N(1985)}{1990 - 1985} = \dfrac{19.3 - 17.0}{5} = 0.46$.

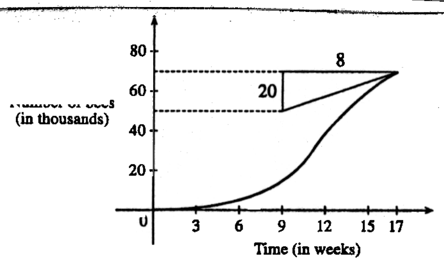
Then $N'(1985) = \lim\limits_{t \to 1985} \dfrac{N(t) - N(1985)}{t - 1985} \approx \dfrac{A + B}{2} = 0.43$ million/year. So

$N(1984) \approx N(1985) + N'(1985)(1984 - 1985) \approx 17.0 + 0.43(-1) = 16.57$ million.

$N'(2000) \approx \dfrac{N(1995) - N(2000)}{1995 - 2000} = \dfrac{22.0 - 24.9}{-5} = 0.58$ million/year.

$N(2006) \approx N(2000) + N'(2000)(2006 - 2000) \approx 24.9 + 0.58(6) = 28.38$ million.

12. (a)



From the figure,

$P'(17) \approx \frac{20}{8} = 2.5$ thousand bees/week.

$P(18) \approx P(17) + P'(17)(18 - 17)$

$\approx 70 + 2.5(1) = 72.5$ or 72,500 bees.

$P(20) \approx P(17) + P'(17)(20 - 17)$

$\approx 70 + 2.5(3) = 77.5$ or 77,500 bees.

(b) Since the tangent line at $t = 17$ is above the graph, our predictions are overestimates.

(c) $P(18)$ is more accurate than $P(20)$ since it is closer to the given data.