Math 1A Fall 2001: Section 3.1 Solutions

6. $g(x) = 5x^8 - 2x^5 + 6 \Rightarrow g'(x) = 5(8x^{8-1}) - 2(5x^{5-1}) + 0 = 40x^7 - 10x^4$

8. $y = 5e^x + 3 \Rightarrow y' = 5(e^x) + 0 = 5e^x$

12. $R(x) = \dfrac{\sqrt{10}}{x^7} = \sqrt{10}\,x^{-7} \Rightarrow R'(x) = -7\sqrt{10}\,x^{-8} = -\dfrac{7\sqrt{10}}{x^8}$

18. $y = \dfrac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2} \Rightarrow y' = 1 - 2(-\frac{1}{2})x^{-3/2} = 1 + 1/(x\sqrt{x})$

22. $u = \sqrt[3]{t^2} + 2\sqrt{t^3} = t^{2/3} + 2t^{3/2} \Rightarrow u' = \frac{2}{3}t^{-1/3} + 2\left(\frac{3}{2}\right)t^{1/2} = \dfrac{2}{3\sqrt[3]{t}} + 3\sqrt{t}$

46. $f(x) = 2x^3 - 3x^2 - 6x + 87$ has a horizontal tangent when $f'(x) = 6x^2 - 6x - 6 = 0 \Leftrightarrow x^2 - x - 1 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{5}}{2}$.

50. If $y = x^2 + x$, then $y' = 2x + 1$. If the point at which a tangent meets the parabola is $(a, a^2 + a)$, then the slope of the tangent is $2a + 1$. But since it passes through $(2, -3)$, the slope must also be $\dfrac{\Delta y}{\Delta x} = \dfrac{a^2 + a + 3}{a - 2}$. Therefore, $2a + 1 = \dfrac{a^2 + a + 3}{a - 2}$. Solving this equation for a we get $a^2 + a + 3 = 2a^2 - 3a - 2 \Leftrightarrow a^2 - 4a - 5 = (a - 5)(a + 1) = 0 \Leftrightarrow a = 5$ or -1. If $a = -1$, the point is $(-1, 0)$ and the slope is -1, so the equation is $y - 0 = (-1)(x + 1)$ or $y = -x - 1$. If $a = 5$, the point is $(5, 30)$ and the slope is 11, so the equation is $y - 30 = 11(x - 5)$ or $y = 11x - 25$.

62. (a) $xy = c \Rightarrow y = \dfrac{c}{x}$. Let $P = \left(a, \dfrac{c}{a}\right)$. The slope of the tangent line at $x = a$ is $y'(a) = -\dfrac{c}{a^2}$. Its equation is $y - \dfrac{c}{a} = -\dfrac{c}{a^2}(x - a)$ or $y = -\dfrac{c}{a^2}x + \dfrac{2c}{a}$, so its y-intercept is $\dfrac{2c}{a}$. Setting $y = 0$ gives $x = 2a$, so the x-intercept is $2a$. The midpoint of the line segment joining $\left(0, \dfrac{2c}{a}\right)$ and $(2a, 0)$ is $\left(a, \dfrac{c}{a}\right) = P$.