Math 1A Fall 2001: Section 3.4 Solutions

4. $g(t) = 4\sec t + \tan t \quad \Rightarrow \quad g'(t) = 4\sec t\tan t + \sec^2 t$

8. $y = \dfrac{\sin x}{1+\cos x} \quad \Rightarrow$

$$\frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x\,(-\sin x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{\cos x + 1}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

12. $y = \csc\theta\,(\theta + \cot\theta) \quad \Rightarrow$

$$y' = \csc\theta\left(1 - \csc^2\theta\right) + (\theta + \cot\theta)(-\csc\theta\cot\theta) = \csc\theta\left(1 - \csc^2\theta - \theta\cot\theta - \cot^2\theta\right)$$

$$= \csc\theta\left(-\cot^2\theta - \theta\cot\theta - \cot^2\theta\right) \qquad \left[1 + \cot^2\theta = \csc^2\theta\right]$$

$$= \csc\theta\left(-\theta\cot\theta - 2\cot^2\theta\right) = -\csc\theta\cot\theta\,(\theta + 2\cot\theta)$$

14. $\dfrac{d}{dx}(\sec x) = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x} = \sec x\tan x$

18. $y = e^x\cos x \quad \Rightarrow \quad y' = e^x(-\sin x) + (\cos x)e^x = e^x(\cos x - \sin x) \quad \Rightarrow$ the slope of the tangent line at $(0,1)$ is $e^0(\cos 0 - \sin 0) = 1(1-0) = 1$ and an equation is $y - 1 = 1(x - 0)$ or $y = x + 1$.

26. $y = \dfrac{\cos x}{2 + \sin x} \quad \Rightarrow$

$$y' = \frac{(2+\sin x)(-\sin x) - \cos x\cos x}{(2+\sin x)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2} = \frac{-2\sin x - 1}{(2+\sin x)^2} = 0 \text{ when}$$

$-2\sin x - 1 = 0 \quad \Leftrightarrow \quad \sin x = -\frac{1}{2} \quad \Leftrightarrow \quad x = \frac{11\pi}{6} + 2\pi n$ or $x = \frac{7\pi}{6} + 2\pi n$, n an integer. So $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$ and the points on the curve with horizontal tangents are: $\left(\frac{11\pi}{6} + 2\pi n, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}}\right)$, n an integer.

38. $$\lim_{x\to 0} x\cot x = \lim_{x\to 0} x\cdot\frac{\cos x}{\sin x} = \lim_{x\to 0}\frac{x\cos x}{\sin x} = \lim_{x\to 0}\frac{\dfrac{x\cos x}{x}}{\dfrac{\sin x}{x}} = \lim_{x\to 0}\frac{\dfrac{\cos x}{1}}{\dfrac{\sin x}{x}} = \frac{\lim\limits_{x\to 0}\cos x}{\lim\limits_{x\to 0}\dfrac{\sin x}{x}} = \frac{1}{1} = 1$$

42. Let $|PR| = x$. Then we get the following formulas for r and h in terms of θ and x:

$\sin\dfrac{\theta}{2} = \dfrac{r}{x} \quad \Rightarrow \quad r = x\sin\dfrac{\theta}{2}$ and $\cos\dfrac{\theta}{2} = \dfrac{h}{x} \quad \Rightarrow \quad h = x\cos\dfrac{\theta}{2}$. Now

$A(\theta) = \frac{1}{2}\pi r^2$ and $B(\theta) = \frac{1}{2}(2r)h = rh$. So

$$\lim_{\theta\to 0+}\frac{A(\theta)}{B(\theta)} = \lim_{\theta\to 0+}\frac{\frac{1}{2}\pi r^2}{rh} = \frac{1}{2}\pi\lim_{\theta\to 0+}\frac{r}{h} = \frac{1}{2}\pi\lim_{\theta\to 0+}\frac{x\sin(\theta/2)}{x\cos(\theta/2)}$$

$$= \frac{1}{2}\pi\lim_{\theta\to 0+}\tan(\theta/2) = 0.$$