4.2 Maximum and Minimum Values

1. A function f has an **absolute minimum** at $x = c$ if $f(c)$ is the smallest function value on the entire domain of f, whereas f has a **local minimum** at c if $f(c)$ is the smallest function value when x is near c.

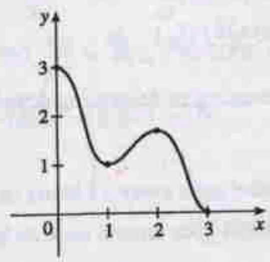
2. (a) The Extreme Value Theorem

(b) See the Closed Interval Method.

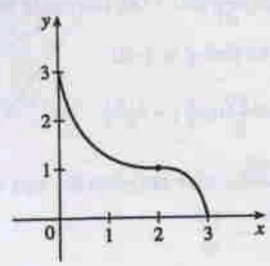
3. Absolute maximum at b; absolute minimum at d; local maxima at b and e; local minima at d and s; neither a maximum nor a minimum at a, c, r, and t.

6. Absolute maximum value is $f(7) = 5$; absolute minimum value is $f(1) = 0$; local maximum values are $f(0) = 2$, $f(3) = 4$, and $f(5) = 3$; local minimum values are $f(1) = 0$, $f(4) = 2$, and $f(6) = 1$.

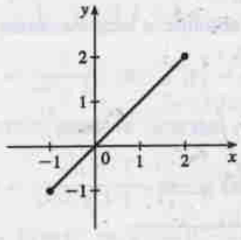
7. The highest point must occur at $x = 0$ and the lowest point must occur at $x = 3$.



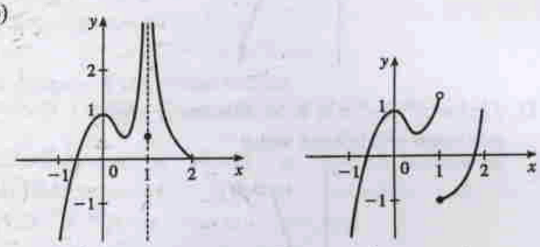
9. The figure has $f'(2) = 0$, so 2 is a critical number. There is an absolute maximum and an absolute minimum, but f has no *local* maximum or minimum.



12. (a) Note that a local maximum cannot occur at an endpoint.



(b)



Note: By the Extreme Value Theorem, f must *not* be continuous.

39. $f(x) = x^2 + \dfrac{2}{x}$, $\left[\frac{1}{2}, 2\right]$. $f'(x) = 2x - \dfrac{2}{x^2} = 2\dfrac{x^3 - 1}{x^2} = 0 \iff x^3 - 1 = 0 \iff (x-1)(x^2+x+1) = 0$, but $x^2 + x + 1 \neq 0$, so $x = 1$. The denominator is 0 at $x = 0$, but not in the desired interval. $f\left(\frac{1}{2}\right) = \frac{17}{4} = 4.25$, $f(1) = 3$, $f(2) = 5$. So $f(2) = 5$ is the absolute maximum and $f(1) = 3$ is the absolute minimum.