2.2

sufficiently close to 2 (but $x \neq 2$).] Yes, the graph

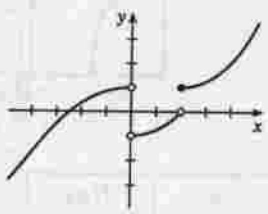
2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 7. No, the limit does not exist because the left- and right-hand limits are different.

8. $\lim_{t \to 12^-} f(t) = 150$ mg and $\lim_{t \to 12^+} f(t) = 300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t = 12$ h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

10. $\lim_{x \to 0^-} f(x) = 1, \quad \lim_{x \to 0^+} f(x) = -1,$

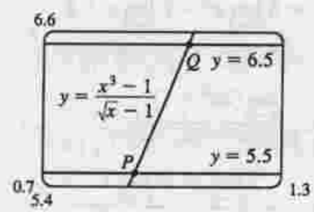
$\lim_{x \to 2^-} f(x) = 0, \ \lim_{x \to 2^+} f(x) = 1, \quad f(2) = 1,$

$f(0)$ is undefined



22. (a) Let $y = (x^3 - 1)/(\sqrt{x} - 1)$.

x	y
0.99	5.92531
0.999	5.99250
0.9999	5.99925
1.01	6.07531
1.001	6.00750
1.0001	6.00075



From the table and the graph, we guess that the limit of y as x approaches 1 is 6.

(b) We need to have $5.5 < \dfrac{x^3 - 1}{\sqrt{x} - 1} < 6.5$. From the graph we obtain the approximate points of intersection $P(0.9313853, 5.5)$ and $Q(1.0649004, 6.5)$. Now $1 - 0.9313853 \approx 0.0686$ and $1.0649004 - 1 \approx 0.0649$, so by requiring that x be within 0.0649 of 1, we ensure that y is within 0.5 of 6.

2.3

2.3 Calculating Limits Using the Limit Laws

1. (a) $\lim_{x\to a}[f(x)+h(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} h(x)$
$= -3 + 8 = 5$

(b) $\lim_{x\to a}[f(x)]^2 = \left[\lim_{x\to a} f(x)\right]^2 = (-3)^2 = 9$

(c) $\lim_{x\to a} \sqrt[3]{h(x)} = \sqrt[3]{\lim_{x\to a} h(x)} = \sqrt[3]{8} = 2$

(d) $\lim_{x\to a} \dfrac{1}{f(x)} = \dfrac{1}{\lim_{x\to a} f(x)} = \dfrac{1}{-3} = -\dfrac{1}{3}$

(e) $\lim_{x\to a} \dfrac{f(x)}{h(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} h(x)} = \dfrac{-3}{8} = -\dfrac{3}{8}$

(f) $\lim_{x\to a} \dfrac{g(x)}{f(x)} = \dfrac{\lim_{x\to a} g(x)}{\lim_{x\to a} f(x)} = \dfrac{0}{-3} = 0$

(g) The limit does not exist, since $\lim_{x\to a} g(x) = 0$ but $\lim_{x\to a} f(x) \neq 0$.

(h) $\lim_{x\to a} \dfrac{2f(x)}{h(x)-f(x)} = \dfrac{2\lim_{x\to a} f(x)}{\lim_{x\to a} h(x) - \lim_{x\to a} f(x)} = \dfrac{2(-3)}{8-(-3)} = -\dfrac{6}{11}$

2. (a) $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2 + 0 = 2$

(b) $\lim_{x\to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim_{x\to 0}[f(x)g(x)] = \lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x) = 0 \cdot 1.3 = 0$

(d) Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, the given limit does not exist.

(e) $\lim_{x\to 2} x^3 f(x) = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3 \cdot 2 = 16$

(f) $\lim_{x\to 1} \sqrt{3+f(x)} = \sqrt{3 + \lim_{x\to 1} f(x)} = \sqrt{3+1} = 2$

5.
$$\begin{aligned}
\lim_{t\to -2}(t+1)^9(t^2-1) &= \lim_{t\to -2}(t+1)^9 \lim_{t\to -2}(t^2-1) && (4)\\
&= \left[\lim_{t\to -2}(t+1)\right]^9 \lim_{t\to -2}(t^2-1) && (6)\\
&= \left[\lim_{t\to -2} t + \lim_{t\to -2} 1\right]^9 \left[\lim_{t\to -2} t^2 - \lim_{t\to -2} 1\right] && (1\ \&\ 2)\\
&= [(-2)+1]^9\left[(-2)^2-1\right] = -3 && (8, 7\ \&\ 9)
\end{aligned}$$

8. (a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \to 2$, just as in Example 3. Remember that in finding $\lim_{x\to a} f(x)$, we never consider $x = a$.

14.
$$\begin{aligned}
\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h} &= \lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}\cdot\frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0}\frac{(1+h)-1}{h\left(\sqrt{1+h}+1\right)} = \lim_{h\to 0}\frac{h}{h\left(\sqrt{1+h}+1\right)}\\
&= \lim_{h\to 0}\frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}
\end{aligned}$$

18.
$$\begin{aligned}
\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h} &= \lim_{h\to 0}\frac{\dfrac{1}{3+h}-\dfrac{1}{3}}{h} = \lim_{h\to 0}\frac{3-(3+h)}{h(3+h)3} = \lim_{h\to 0}\frac{-h}{h(3+h)3}\\
&= \lim_{h\to 0}\left[-\frac{1}{3(3+h)}\right] = -\frac{1}{\lim_{h\to 0}[3(3+h)]} = -\frac{1}{3(3+0)} = -\frac{1}{9}
\end{aligned}$$

30. If $x > 2$, then $|x-2| = x-2$, so $\lim_{x\to 2^+} \frac{|x-2|}{x-2} = \lim_{x\to 2^+} \frac{x-2}{x-2} = \lim_{x\to 2^+} 1 = 1$. If $x < 2$, then $|x-2| = -(x-2)$, so $\lim_{x\to 2^-} \frac{|x-2|}{x-2} = \lim_{x\to 2^-} \frac{-(x-2)}{x-2} = \lim_{x\to 2^-} -1 = -1$. The right and left limits are different, so $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist.

42. Let $f(x) = H(x)$ and $g(x) = 1 - H(x)$, where H is the Heaviside function defined in Exercise 1.3.53. Thus, either f or g is 0 for any value of x. Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} [f(x)g(x)] = \lim_{x\to 0} 0 = 0$.