3.2

2. Quotient Rule: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \dfrac{x - 3x^{3/2}}{x^{1/2}} \Rightarrow$

$$F'(x) = \frac{x^{1/2}\left(1 - \frac{9}{2}x^{1/2}\right) - \left(x - 3x^{3/2}\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}\right)^2}$$

$$= \frac{x^{1/2} - \frac{9}{2}x - \frac{1}{2}x^{1/2} + \frac{3}{2}x}{x} = \frac{\frac{1}{2}x^{1/2} - 3x}{x} = \frac{1}{2}x^{-1/2} - 3$$

Simplifying first: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \sqrt{x} - 3x = x^{1/2} - 3x \Rightarrow F'(x) = \frac{1}{2}x^{-1/2} - 3$ (equivalent)

For this problem, simplifying first seems to be the better method.

3. By the Product Rule, $f(x)$ ...

4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \Rightarrow g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

8. $f(u) = \dfrac{1-u^2}{1+u^2} \Rightarrow f'(u) = \dfrac{(1+u^2)(-2u) - (1-u^2)(2u)}{(1+u^2)^2} = \dfrac{-2u - 2u^3 - 2u + 2u^3}{(1+u^2)^2} = -\dfrac{4u}{(1+u^2)^2}$

10. $H(t) = e^t(1 + 3t^2 + 5t^4) \Rightarrow$

$H'(t) = e^t(6t + 20t^3) + (1 + 3t^2 + 5t^4)e^t = e^t(5t^4 + 20t^3 + 3t^2 + 6t + 1)$

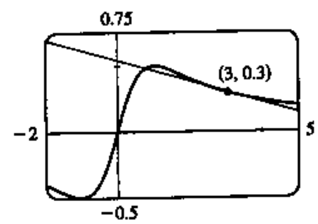
20. $y = \dfrac{\sqrt{x}}{x+1} \Rightarrow y' = \dfrac{(x+1)\left(\frac{1}{2\sqrt{x}}\right) - \sqrt{x}(1)}{(x+1)^2} = \dfrac{(x+1) - (2x)}{2\sqrt{x}(x+1)^2} = \dfrac{1-x}{2\sqrt{x}(x+1)^2}$. At $(4, 0.4)$,

$y' = \frac{-3}{100} = -0.03$, and an equation of the tangent line is $y - 0.4 = -0.03(x-4)$, or $y = -0.03x + 0.52$.

22. (a) $y = f(x) = \dfrac{x}{1+x^2} \Rightarrow$

$f'(x) = \dfrac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $(3, 0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y - 0.3 = -0.08(x-3)$ or $y = -0.08x + 0.54$.

(b)



28. We are given that $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$.

(a) $(f+g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$

(b) $(fg)'(3) = f(3)g'(3) + g(3)f'(3) = (4)(5) + (2)(-6) = 20 - 12 = 8$

(c) $\left(\dfrac{f}{g}\right)'(3) = \dfrac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2} = \dfrac{(2)(-6) - (4)(5)}{(2)^2} = \dfrac{-32}{4} = -8$

(d) $\left(\dfrac{f}{f-g}\right)'(3) = \dfrac{[f(3) - g(3)]\,f'(3) - f(3)\,[f'(3) - g'(3)]}{[f(3) - g(3)]^2}$

$$= \frac{(4-2)(-6) - 4(-6-5)}{(4-2)^2} = \frac{-12+44}{2^2} = 8$$

34. (a) $f(20) = 10{,}000$ means that when the price of the fabric is \$20/yard, 10,000 yards will be sold. $f'(20) = -350$ means that as the price of the fabric increases past \$20/yard, the amount of fabric which will be sold is decreasing at a rate of 350 yards per (dollar per yard).

(b) $R(p) = pf(p) \quad \Rightarrow \quad R'(p) = pf'(p) + f(p) \cdot 1 \quad \Rightarrow$
$R'(20) = 20f'(20) + f(20) \cdot 1 = 20(-350) + 10{,}000 = 3000$. This means that as the price of the fabric increases past \$20/yard, the total revenue is increasing at \$3000/(\$/yard). Note that the Product Rule indicates that we will lose \$7000/(\$/yard) due to selling less fabric, but that that loss is more than made up for by the additional revenue due to the increase in price.