vi) [redacted] First check if abs. conv. let $u = \ln(\ln(x))$, $du = \frac{dx}{x \cdot \ln(x)}$.

So $\int \frac{\ln(\ln(x))}{x \ln(x)} dx = \int u \, du = \frac{u^2}{2}$

So $\int_3^\infty \frac{\ln(\ln(x))}{x \cdot \ln(x)} dx = \lim_{b \to \infty} \left. \frac{[\ln(\ln(x))]^2}{2} \right|_3^b = \lim_{b \to \infty} \frac{\ln(\ln(b))^2}{2} - \frac{\ln(\ln(3))^2}{2} = \infty$

Not abs. Conv.

Use AST to check for cond. conv.

a) $\lim_{x \to \infty} \frac{\ln(\ln(x))}{x \cdot \ln(x)} \underset{\text{l'Hopital}}{=} \lim_{x \to \infty} \frac{\frac{1}{x \cdot \ln(x)}}{1 + \ln(x)} = \lim_{x \to \infty} \frac{1}{x \cdot \ln(x) \cdot (1 + \ln(x))} = 0$

b) $f(x) = \frac{\ln(\ln(x))}{x \cdot \ln(x)}$; $f'(x) = \frac{x \cdot \ln(x) \cdot \frac{1}{x \ln(x)} - \ln(\ln(x)) \cdot (1 + \ln(x))}{x^2 (\ln(x))^2}$

$f'(x) < 0$ when $\ln(\ln(x)) \cdot (1 + \ln(x)) > 1$

this is true when $\ln(\ln(x)) \geq 1$ or $\ln(x) \geq e$ or $x \geq e^e$

So by Alt. Series test, $f(x)$ is eventually decreasing.

Hence $\boxed{\text{Cond. Conv.}}$

3) Find Radius of Conv. by Ratio Test

$$L = \lim_{k \to \infty} \left| \frac{(-1)^{k+2} \cdot 1 \cdot 3 \cdots (2k+1) \cdot (x-5)^{k+1}}{\sqrt{k} \, 4^{k+1} \cdot (k+2)!} \right| \left| \frac{(-1)^{k+1} \sqrt{k-1} \cdot 4^k (k+1)!}{1 \cdot 3 \cdots (2k-1) \cdot (x-5)^k} \right|$$

$$= \lim_{k \to \infty} \frac{(2k+1)}{4(k+2)} \sqrt{\frac{k-1}{k}} \, |x-5| = \frac{|x-5|}{2} < 1 \Rightarrow |x-5| < 2$$

$\Rightarrow -2 < x-5 < 2$

$\Rightarrow 3 < x < 7$

Check endpts:

@ $x=3$, have $\sum \frac{(-1)^{k+1}(-2)^k \cdot 1 \cdot 3 \cdot 5 \cdots (2k-1)}{\sqrt{k-1} \cdot 4^k \cdot (k+1)!} = \sum \frac{(-1)^{2k+1} \cdot 1 \cdot 3 \cdot 5 \cdots (2k-1)}{\sqrt{k-1} \, 2^k \, (k+1)!}$

note: this series is <u>not</u> alternating since $2k+1$ is always odd.

now $\sum \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{\sqrt{k-1} \cdot 2^k \cdot (k+1)!} = \sum \frac{1}{\sqrt{k-1}} \cdot \underbrace{\frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots 2k}}_{\text{this is } < 1} \cdot \frac{1}{(k+1)}$

So compare our series to $\sum \frac{1}{\sqrt{k-1}} \cdot \frac{1}{k+1}$

Use limit comparison of new series to $\sum \frac{1}{k^{3/2}}$

This last converges since p-series with $p = 3/2 > 1$.

So original series converges.

@ $x=7$, this is alternating version of series at $x=3$, so also converges

So $\boxed{\text{Interval of Convergence is } [3,7]}$