SECTION 11.9 REPRESENTATIONS OF FUNCTIONS AS POWER SERIES

- If the power series $\sum c_n(x-a)^n$ has a radius of convergence $R>0$, then the function f defined by $f(x)=\sum_{n=0}^{\infty} c_n(x-a)^n$ is differentiable on the interval $(a-R, a+R)$ and

(i) $f'(x)=c_1+2c_2(x-a)+3c_3(x-a)^2+\ldots=\sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$

(ii) $\int f(x)dx = C+c_0(x-a)+c_1\frac{(x-a)^2}{2}+c_2\frac{(x-a)^3}{3}+\ldots = C+\sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$

- Review this section to represent certain types of functions as sums of power series by manipulating geometric series or by differentiating or integrating it

SECTION 11.10 TAYLOR AND MACLAURIN SERIES

- If f has a power series representation at a, that is, if $f(x)=\sum_{n=0}^{\infty} c_n(x-a)^n \quad |x-a|<R$, then coefficients are $c_n=\frac{f^{(n)}(a)}{n!}$

* $\boxed{f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n}$ is the new power series representation at a. Taylor Series

* $\boxed{f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n = f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\ldots}$ is the representation at $a=0$ Maclaurin Series

Thus, if f can be represented as a power series about a, then f is equal to the sum of its Taylor series. The Maclaurin Series is a special case where $a=0$.

- If $f(x)=T_n(x)+R_n(x)$, where T_n is the n^{th}-degree Taylor polynomial of f at a and $\lim_{n\to\infty} R_n(x)=0$ for $|x-a|<R$, then f is equal to the sum of its Taylor series on $|x-a|<R$

- Taylor's Inequality. If $|f^{(n+1)}(x)|\le M$ for $|x-a|\le d$, then the remainder $R_n(x)$ of the Taylor series satisfies $|R_n(x)|\le\frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a|\le d$.

- $\boxed{\lim_{n\to\infty}\frac{x^n}{n!}=0}$ $\boxed{e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}}$ If $x=1$, then $\boxed{e=\sum_{n=0}^{\infty}\frac{1}{n!}}$

Maclaurin Expansion of $\boxed{\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}}$ and $\boxed{\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}}$

$\boxed{\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n}$ and $\boxed{\tan^{-1}x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}}$

Study chart on page 758 of Important Maclaurin Series