Math 1b HW Solutions (Week 1)

7.4

② (a) $\sum_{k=1}^{4} k\sin\frac{k\pi}{2} = 1\cdot\sin\frac{\pi}{2} + 2\cdot\sin\frac{2\pi}{2} + 3\cdot\sin\frac{3\pi}{2} + 4\cdot\sin\frac{4\pi}{2} = 1+0+(-3)+0 = \boxed{-2}$

(b) $\sum_{j=0}^{5}(-1)^j = (-1)^0+(-1)^1+(-1)^2+(-1)^3+(-1)^4+(-1)^5 = 1-1+1-1+1-1 = \boxed{0}$

(c) $\sum_{i=7}^{20} e^2 = (20-7+1)e^2 = \boxed{14e^2}$

(d) $\sum_{m=3}^{5} 2^{m+1} = 2^{3+1}+2^{4+1}+2^{5+1} = 16+32+64 = \boxed{112}$

(e) $\sum_{n=1}^{6} \ln n = \ln 1+\ln 2+\ln 3+\ldots+\ln 6 = \ln 6! = \boxed{\ln 720}$

(f) $\sum_{k=0}^{10} \cos k\pi = \cos 0+\cos\pi+\cos 2\pi+\ldots\cos 10\pi = 1+(-1)+1+(-1)+\ldots+(-1)+1 = \boxed{1}$

④ $3\cdot1+3\cdot2+3\cdot3+\ldots 3\cdot20 = \boxed{\sum_{k=1}^{20} 3k}$

⑧ $1+3+5+7+\ldots+15 = \boxed{\sum_{k=0}^{7} 2k+1 = \sum_{n=1}^{8} 2n-1}$

⑯ $\sum_{k=1}^{100}(2^{k+1}-2^k) = (2^2-2)+(2^3-2^2)+(2^4-2^3)+\ldots+(2^{101}-2^{100}) = -2+2^{101} = \boxed{2^{101}-2}$

11.3

③ $\sum_{k=1}^{\infty}\left(\frac{-3}{4}\right)^{k-1} = \frac{1}{1-(-3/4)} = 4/7$ Geometric series w/ $a=1$ $r=-3/4$

④ $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^{k+2} = \frac{(2/3)^3}{1-(2/3)} = 8/9$ Geometric series w/ $a=(2/3)^3=8/27$ $r=2/3$

⑥ $\sum_{k=1}^{\infty}\left(-\frac{3}{2}\right)^{k+1}$ diverges Geometric series w/ $r=-3/2$ $(|r|\geq 1)$

⑬ $\sum_{k=1}^{\infty}\frac{4^{k+2}}{7^{k-1}} = \frac{4^3}{1}+\frac{4^4}{7}+\frac{4^5}{7^2}+\ldots = 4^3+4^3\left(\frac{4}{7}\right)+4^3\left(\frac{4}{7}\right)^2+\ldots = \frac{4^3}{1-4/7} = \frac{448}{3}$ Geometric $r=4/7$ $a=64$

⑭ $\sum_{k=1}^{\infty} 5^{3k}7^{-k} = \sum_{k=1}^{\infty} 5^{3k}7^{-(k-1)} = \sum_{k=1}^{\infty}\frac{5^{3k}}{7^{k-1}} = \sum_{k=1}^{\infty} 5^3\frac{5^{3(k-1)}}{7^{k-1}}$ diverges Geometric $a=125$ $r=\frac{125}{7}$

11.3 cont'd

(20) $0.45114141 4\ldots = 0.451 + 0.00014 + 0.0000014 + \ldots = .451 + \frac{.00014}{1-\frac{1}{100}} = \frac{44663}{99000}$

OR Let $x = .451\overline{14}$

$100x = 45.114\overline{14}$ $\Rightarrow 99x = 44.663 \Rightarrow x = \frac{44663}{99000}$

(24) Total volume $= \sum_{k=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^k\right]^3 = 1 + \left(\tfrac{1}{2}\right)^3 + \left(\tfrac{1}{2}\right)^6 + \left(\tfrac{1}{2}\right)^9 + \ldots = \frac{1}{1-1/8} = 8/7$

(26) (a) $\sum_{k=0}^{\infty} (-1)^k x^k = \sum_{k=0}^{\infty} (-x)^k$ since $|-x| < 1$, it's a convergent geometric series with $a=1$ $r=-x$

$= \frac{1}{1-(-x)} = \frac{1}{1+x}$

(b) $\sum_{k=0}^{\infty} (x-3)^k$ = a geometric series with $a=1$, $r = x-3$. since $2 < x < 4$, $|r| < 1$ so the series converges

$= \frac{1}{1-(x-3)} = \frac{1}{4-x}$

(c) $\sum_{k=0}^{\infty} (-1)^k x^{2k} = 1 - x^2 + x^4 - x^6 + \ldots$ = Geometric series w/ $a=1$, $r = -x^2$

since $-1 < x < 1$, $|r| = |-x^2| < 1$ so the series converges

$= \frac{1}{1-(-x^2)} = \frac{1}{1+x^2}$

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(2)(b) $\sum_{k=1}^{\infty} \left[7^{-k} 3^{k+1} - \frac{2^{k+1}}{5^k}\right] = \sum_{k=1}^{\infty} 3 \cdot \frac{3^k}{7^k} - \sum_{k=1}^{\infty} 2\left(\frac{2}{5}\right)^k = \frac{9/7}{1-3/7} - \frac{4/5}{1-2/5} = \frac{9}{4} - \frac{4}{3} = \frac{11}{12}$

(4) (a) $\sum_{k=1}^{\infty} k^{-4/3} = \sum_{k=1}^{\infty} \frac{1}{k^{4/3}}$ Converges

(b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k}} = \sum_{k=1}^{\infty} \frac{1}{k^{1/4}}$ diverges

by p-series test

(c) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^5}} = \sum_{k=1}^{\infty} \frac{1}{k^{5/3}}$ converges

(d) $\sum_{k=1}^{\infty} \frac{1}{k^{\pi}}$ $\pi > 1$ converges

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(6) (a) $\lim_{k\to\infty} \frac{k}{e^k} = \lim_{k\to\infty} \frac{1}{e^k} = 0$ So it may converge more tests needed (inconclusive)

(b) $\lim_{k\to\infty} \ln k = \infty$ so $\sum_{k=1}^{\infty} \ln k$ diverges

(c) $\lim_{k\to\infty} \frac{1}{\sqrt{k}} = 0$ so divergence limit test is inconclusive

(d) $\lim_{k\to\infty} \frac{\sqrt{k}}{\sqrt{k}+3} = 1$ so series diverges

(10) $\sum_{k=1}^{\infty} \frac{3}{5k} = \frac{3}{5}\sum_{k=1}^{\infty}\frac{1}{k}$ harmonic series diverges so this series also diverges

(17) $\sum_{k=1}^{\infty} (1+\frac{1}{k})^{-k}$ $\lim_{k\to\infty} (1+\frac{1}{k})^{-k} = \frac{1}{e}$ so series diverges.

(23) $\sum_{k=5}^{\infty} 7k^{-1.01} = 7\cdot\sum_{k=5}^{\infty}\frac{1}{k^{1.01}}$ p-series test $p=1.01>1$ so series converges

(note $7\sum_{k=1}^{\infty}\frac{1}{1.01^k}$ converges and $7\cdot\sum_{k=1}^{\infty}\frac{1}{k^{1.01}} > 7\sum_{k=5}^{\infty}\frac{1}{k^{1.01}}$)