Math 1b (Week 6) HW Solutions

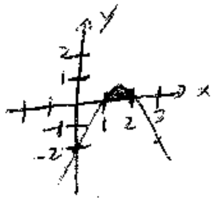
7.6

(6) $\text{Area} = \int_1^4 x^{-3/5}\,dx = \left.\dfrac{x^{2/5}}{2/5}\right|_1^4 = \dfrac{5}{2}\left(4^{2/5}-1\right)$

(10) $\int_{-1}^{2} x(1+x^3)\,dx = \int_{-1}^{2} x\,dx + \int_{-1}^{2} x^4\,dx = \left(\dfrac{2^2}{2}-\dfrac{(-1)^2}{2}\right) + \left(\dfrac{2^5}{5}-\dfrac{(-1)^5}{5}\right) = \dfrac{3}{2}+\dfrac{33}{5} = \dfrac{81}{10}$

(12) $\int_1^2 \dfrac{1}{x^6}\,dx = \int_1^2 x^{-6}\,dx = \left.\dfrac{x^{-5}}{-5}\right|_1^2 = \dfrac{-1}{5}\left(\dfrac{1}{32}-1\right) = \dfrac{31}{160}$

(36)



$\int_1^2 (1-x)(x-2)\,dx = \int_1^2 -x^2+3x-2\,dx = \left.-\dfrac{x^3}{3}+\dfrac{3x^2}{2}-2x\right|_1^2$

$= \left(\dfrac{-8}{3}+6-4\right) - \left(-\dfrac{1}{3}+\dfrac{3}{2}-2\right) = \dfrac{1}{6}$

7.3

(2) (a) $\int \sec^2(4x+1)\,dx = \dfrac{1}{4}\int \sec^2 u\,du = \dfrac{1}{4}\tan u + C = \dfrac{1}{4}\tan(4x+1)+C$

$u = 4x+1 \Rightarrow du = 4dx$

(b) $\int y\sqrt{1+2y^2}\,dy = \dfrac{1}{4}\int \sqrt{u}\,du = \dfrac{1}{4}\cdot\dfrac{2u^{3/2}}{3}+C = \dfrac{(1+2y^2)^{3/2}}{6}+C$

$u = 1+2y^2 \quad du = 4y\,dy$

(c) $\int \sqrt{\sin\pi\theta}\,\cos\pi\theta\,d\theta = \dfrac{1}{\pi}\int\sqrt{u}\,du = \dfrac{2}{3\pi}u^{3/2}+C = \dfrac{2(\sin\pi\theta)^{3/2}}{3\pi}+C$

$u = \sin\pi\theta \Rightarrow du = \pi\cos\pi\theta\,d\theta$

(d) $\int (2x+7)(x^2+7x+3)^{4/5}\,dx = \int u^{4/5}\,du = \dfrac{5}{9}u^{9/5}+C = \dfrac{5}{9}(x^2+7x+3)^{9/5}+C$

$u = x^2+7x+3 \Rightarrow du = (2x+7)dx$

(e) $\int \dfrac{e^x}{1+e^x}\,dx = \int \dfrac{du}{u} = \ln u + C = \ln(1+e^x)+C$

$u = 1+e^x \Rightarrow du = e^x\,dx$

(8) $\int (3x-1)^5\,dx = \dfrac{1}{3}\int u^5\,du = \dfrac{1}{3}\,\dfrac{u^6}{6}+C = \dfrac{(3x-1)^6}{18}+C$

(10) $\int \sin 3x\,dx = \dfrac{1}{3}\int \sin u\,du = \dfrac{-1}{3}\cos u + C = -\dfrac{1}{3}\cos 3x + C$

7.3

(14) $\int \frac{x}{\sqrt{4-5x^2}}\,dx = \frac{-1}{10}\int \frac{du}{\sqrt{u}} = \frac{-1}{10}\cdot 2\sqrt{u}+C = \frac{-1}{5}\sqrt{4-5x^2}+C$

$u=4-5x^2 \Rightarrow du=-10x\,dx$

(16) $\int \frac{1}{(1-3x)^2}\,dx = \frac{-1}{3}\int u^{-2}\,du = \frac{-1}{3}\,\frac{u^{-1}}{-1}+C = \frac{1}{3(1-3x)}+C$

(20) $\int x^3 e^{x^4}\,dx = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^{x^4}+C$

(28) $\int \sqrt{e^x}\,dx = \int e^{x/2}\,dx = 2\int e^u\,du = 2e^{x/2}+C$

(50) Method 1: $\int (5x-1)^2\,dx = \int 25x^2-10x+1\,dx = \frac{25x^3}{3}-5x^2+x+C$

(a) Method 2: $\int (5x-1)^2\,dx = \frac{1}{5}\int u^2\,du = \frac{u^3}{15}+D = \frac{(5x-1)^3}{15}+D$

(b) The two constants of integration are different.

Note, in particular, that $\frac{(5x-1)^3}{15} = \frac{125x^3-75x^2+15x-1}{15} = \frac{25}{3}x^3-5x^2+x-\frac{1}{15}$

Let $C = D-\frac{1}{15}$ then the values become exactly the same.

8.2

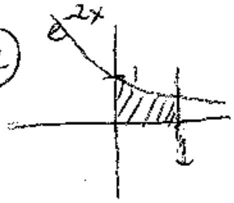
(2) Volume $= \pi\int_0^1 (2-x^2)^2 - x^2\,dx = \pi\int_0^1 (x^4-5x^2+4)\,dx = \pi\left(\frac{1}{5}-\frac{5}{3}+4\right)$

$= \frac{38\pi}{15}$

(4) $V = \pi\int_{1/2}^{2} (2)^2-\left(\frac{1}{y}\right)^2 dy = \pi\int_{1/2}^{2}\left(4-\frac{1}{y^2}\right)dy = \pi\left[4(2-1/2) - \left(\frac{-1}{2}-\frac{-1}{1/2}\right)\right]$

$= \frac{9\pi}{2}$

(12)

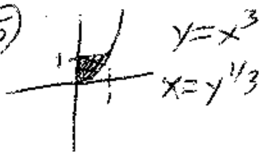


$V = \pi\int_0^1 \left(e^{-2x}\right)^2 dx = \frac{-\pi}{4}\int_a^b e^u\,du = \frac{-\pi}{4}e^{-4x}\Big|_0^1 = \frac{-\pi}{4}\left(\frac{1}{e^4}-1\right)$

$= \frac{\pi}{4}\left(1-\frac{1}{e^4}\right)$

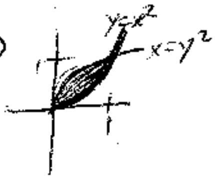
8.2

(15)



$V=\pi\int_0^1 (y^{1/3})^2\,dy = \pi\cdot\frac{y^{5/3}}{5/3}\Big|_0^1 = \frac{3\pi}{5}$

(20)



$V=\pi\int_0^1\left((\sqrt{y})^2-(y^2)^2\right)dy = \pi\int_0^1 y-y^4\,dy = \pi\left(\frac{1}{2}-\frac{1}{5}\right) = \frac{3\pi}{10}$

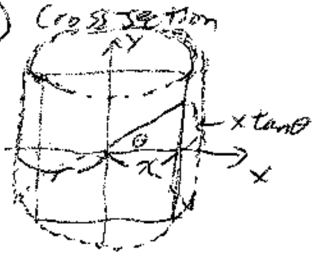
(35)(a) The radius as a function of x is $r(x)=\sqrt{1-x^2}$ because $r(x)=y$

So $V=\frac{\pi}{2}\int_{-1}^{1}(1-x^2)\,dx = \pi\int_0^1(1-x^2)\,dx = \frac{2\pi}{3}$

(b) ½ the edge of a square is $\sqrt{1-x^2}$ so...

$V=\int_{-1}^{1}\left(2\sqrt{1-x^2}\right)^2dx = 8\int_0^1(1-x^2)\,dx = 16/3$

(45)

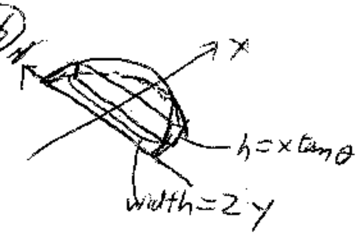


So $V=\int_{-r}^{r}\frac{1}{2}x\cdot x\tan\theta\,dy$ because $x^2=r^2-y^2$

$=\tan\theta\int_0^r(r^2-y^2)\,dy$

$=\tan\theta\cdot\left(r^3-\frac{r^3}{3}\right) = \frac{2r^3\tan\theta}{3}$

(46)



$V=\int_0^r 2yx\tan\theta\,dx$

$=2\tan\theta\int_0^r x\sqrt{r^2-x^2}\,dx$

(let $u=r^2-x^2$) $=\tan\theta\int_a^b\sqrt{u}\,du = -\tan\theta\cdot\frac{2(r^2-x^2)^{3/2}}{3}\Big|_0^r$

$=-\tan\theta\left(0-\frac{2}{3}r^3\right)$

$=\frac{2r^3\tan\theta}{3}$

$\underline{A:}$ $V_x(t) = 2\sin(3t)$ and $S_x(0) = 1$

1. $S_x(t) = \int V_x(t)\,dt = \int 2\sin 3t\,dt = -\frac{2}{3}\cos 3t + C$

$S_x(0) = 1 = -\frac{2}{3} + C$ so $C = \frac{5}{3}$

$x(t) = S_x(t) = -\frac{2}{3}\cos(3t) + \frac{5}{3}$

2. The particle moves to the left when $V_x(t) < 0$

So solve for $V_x(t) = 0$

$2\sin(3t) = 0 \iff \sin 3t = 0 \iff 3t = \pi n$ for some integer n.

$t = \frac{\pi n}{3}$

so $0 \le t \le \frac{2\pi}{3}$ when $n = 0$ or 1 or 2

So $V_x(t) = 0$ when $t = 0, \pi/3$ or $2\pi/3$

Notice that $\sin(3t) < 0$ when $\pi < 3t < 2\pi \iff \pi/3 < t < \frac{2\pi}{3}$

So the particle moves to the left during the interval $(\pi/3, 2\pi/3)$

3. Total distance $= \left|\int_0^{\pi/3} 2\sin 3t\,dt\right| + \left|\int_{\pi/3}^{2\pi/3} 2\sin 3t\,dt\right|$

$= \left|\left(-\frac{2}{3}\cos\pi + \frac{2}{3}\cos 0\right)\right| + \left|-\frac{2}{3}\cos 2\pi + \frac{2}{3}\cos\pi\right|$

$= \left|\left(\frac{2}{3} + \frac{2}{3}\right)\right| + \left|\left(-\frac{2}{3} - \frac{2}{3}\right)\right| = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$