Math 1b HW Solutions (Week 7)

8-4

④ $L=\int_0^1\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$ $\qquad x=\frac{1}{3}(y^2+2)^{3/2}$

$\frac{dx}{dy}=y\sqrt{y^2+2}$ so $1+\left(\frac{dx}{dy}\right)^2=1+y^4+2y^2$
$=(y^2+1)^2$

$L=\int_0^1 y^2+1\,dy=\frac{1}{3}+1=\frac{4}{3}$

⑧ $x=\frac{y^4}{8}+\frac{y^{-2}}{4}$ $\quad y\in[1,4]$ $\qquad \frac{dx}{dy}=\frac{y^3}{2}-\frac{y^{-3}}{2}$

$1+\left(\frac{dx}{dy}\right)^2=\frac{y^6}{4}-\frac{1}{2}+\frac{y^{-6}}{4}+1$
$=\left(\frac{y^3}{2}+\frac{y^{-3}}{2}\right)^2$

$L=\int_1^4\frac{y^3}{2}+\frac{y^{-3}}{2}\,dy$
$=\frac{y^4}{8}-\frac{y^{-2}}{4}\Big|_1^4=\frac{2055}{64}$

⑩ $x=(1+t)^2$ $\quad y=(1+t)^3$ $\qquad \frac{dx}{dt}=2(1+t)$ $\quad \frac{dy}{dt}=3(1+t)^2$

$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2$

$L=\int_0^1(1+t)\sqrt{4+9(1+t)^2}\,dt$ $\qquad =(1+t)^2\left(4+9(1+t)^2\right)$

Let $u=4+9(1+t)^2$ $\quad$ so $L=\frac{1}{18}\int_a^b\sqrt{u}\,du=\frac{1}{27}\left(4+9(1+t)^2\right)^{3/2}\Big|_0^1$

$\Rightarrow du=18(1+t)\,dt$

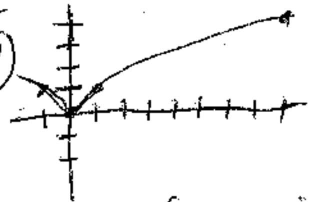
$=\frac{1}{27}\left(40^{3/2}-13^{3/2}\right)$

$=\frac{1}{27}\left(40\sqrt{40}-13\sqrt{13}\right)$

$=\frac{1}{27}\left(80\sqrt{10}-13\sqrt{13}\right)$

8.4

(19)



$\leftarrow$ (a) $y = x^{2/3}$ $\quad \frac{dx}{dy} = \frac{3y^{1/2}}{2}$

$x = y^{3/2}$

(b) The function is ~~is~~ not differentiable at $x=0$ with respect to x (i.e. $\frac{dy}{dx}$ doesn't exist for $x=0$)

(c) $L_{(-1,0)} = \int_{-1}^{0} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy = \int_0^1 \sqrt{1+\frac{9y}{4}}\,dy = \frac{8}{27}\left(\frac{13\sqrt{13}}{8} - 1\right)$

$L_{(0,8)} = \int_0^4 \sqrt{1+\frac{dx}{dy}}\,dy = \int_0^4 \sqrt{1+\frac{9y}{4}}\,dy = \frac{8}{27}\left(10\sqrt{10} - 1\right)$

$L_{TOT} = \frac{8}{27}\left(\frac{13\sqrt{13}}{8} + 10\sqrt{10} - 2\right) = \frac{13\sqrt{13} + 80\sqrt{10} - 16}{27}$

8.5

(2) $S = \int_1^4 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx = 2\pi\int_1^4 \sqrt{x+1/4}\,dx = \frac{(17\sqrt{17} - 5\sqrt{5})\pi}{6}$

(4) $S = \int_1^2 2\pi x^3\sqrt{1+9x^4}\,dx = \frac{2\pi}{36}\int_a^b \sqrt{u}\,du$

$u = 1+9x^4 \quad du = 36x^3 dx$

$= \frac{\pi}{18}\,\frac{2(1+9x^4)^{3/2}}{3}\Big|_1^2 = \frac{\pi}{27}\left[(1+144)^{3/2} - 10^{3/2}\right] = \frac{5\pi(29\sqrt{145} - 2\sqrt{10})}{27}$

(8) $S = \int_{-1}^{0} 2\pi\cdot 2\sqrt{1-y}\cdot\sqrt{1+\frac{1}{1-y}} = 4\pi\int_{-1}^{0}\sqrt{2-y}\,dy = \frac{8\pi(3\sqrt{3} - 2\sqrt{2})}{3}$

(18) $S = \int_{-r}^{r} 2\pi\sqrt{r^2-x^2}\cdot\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx = \int_{-r}^{r} 2\pi r\,dx = 2\pi r(2r) = 4\pi r^2$

8.6

(2) $\text{Work} = \int_0^5 F[x]\,dx$ = area under the force graph

$$= 2\cdot 40 + \tfrac{1}{2}\,3\cdot 40 = 140\,J$$

(4) $F(x) = kx$ (a) so $45\,N = F(.05) = k\cdot .05 \Rightarrow k = 900\,N/m$

(b) $\text{Work} = \int_0^{0.03} 900x\,dx = 0.405\,J$

(c) $\text{Work} = \int_{0.05}^{0.1} 900x\,dx = 3.375\,J$

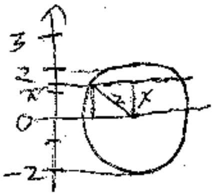
(5cm displaced → 10cm displaced)

(11) Let (x) = height of water in tank.
to move the (x) layer to the top, $9810\cdot(x)\cdot$ area dx needs to go up by $(3-x)$.
The area is given by $6\cdot\frac{4x}{3}$ So $\text{Work} = \int_0^2 (3-x)\cdot 9810\cdot\frac{4x}{3}\cdot 6\,dx$

$$= 78480\int_0^2 3x - x^2\,dx$$

$$= 261{,}600\,J$$

(12) By similar reasons to #11, the liquid at level x has a differential volume of $\underbrace{10}_{\text{length}}\,\underbrace{\pi(4-x^2)}_{\text{cross length}}\cdot\underbrace{50}_{\text{density}}\,\underbrace{dx}_{\text{"thickness"}}$



So $\text{Work} = \int_{-2}^{2} \underbrace{(3-x)}_{\text{distance raised}}\cdot 20\pi(4-x^2)\cdot 50\,dx = 3000\int_{-2}^{2}\sqrt{4-x^2}\,dx$
$$-1000\int_{-2}^{2} x\sqrt{4-x^2}\,dx$$
$$= 6000\pi\ \text{ft}\cdot\text{lb}.$$

(14) Total weight at height $x = \underbrace{3}_{\text{rocket}} + \underbrace{\left[40 - \frac{2x}{1000}\right]}_{\text{fuel}}$

So $\text{Work} = \int_0^{3000} 43 - \frac{x}{500}\,dx = 120{,}000$ ft-tons.

8.6

(17) (a) $150\text{ lbs} = \frac{k}{(4000)^2} \Rightarrow k = 2.4\times10^9 \Rightarrow w(x) = \frac{2400000000}{x^2}$

(b) $6000\text{ lbs} = \frac{k}{(4000)^2} \Rightarrow k = 9.6\times10^{10} \Rightarrow w(x) = \frac{9.6\times10^{10}}{(x+4000)^2}\text{ lbs}$

(c) $W = \int_{4000}^{5000} \frac{9.6\times10^{10}}{x^2}\,dx = 4800{,}000\text{ mi}\cdot\text{lb} = 2.5344\times10^{10}\text{ ft}\cdot\text{lb}.$

8.7

(4) $F = \int_1^3 \underset{N/m^3}{9810}\, x \cdot 4\,dx = 39240 \cdot \left(\frac{3^2-1^2}{2}\right) = 159{,}960\text{ N}.$

(8) $\text{width} = 16 + 2\left(\frac{12-x}{2}\right)$ for $12 \geq x \geq 4$

$= 28 - x$

so

$$F = \int_4^{12} 62.4\text{ lb/ft}^3\,(28-x)\,dx = 77{,}209.6\text{ lb.}$$

(12)

$h(x) = \frac{x\sqrt{3}}{2}$ so $F = \int_0^{100} 9810 \cdot \frac{x\sqrt{3}}{2} \cdot \underbrace{200}_{\text{width}}\,dx$

$= 981000\sqrt{3}\int_0^{100} x\,dx$

$= 4905{,}000\sqrt{3}\text{ N}$