Math 1b HW Solutions (Week 8)

9.2

(2) $\int x e^{3x}\,dx = \frac{x}{3}e^{3x} - \int \frac{1}{3}e^{3x}dx = \frac{xe^{3x}}{3} - \frac{1}{9}e^{3x} + C$

$u = x \quad dv = e^{3x}dx$

$du = dx \quad v = \frac{1}{3}e^{3x}$

(8) $\int x^2 \sin x\,dx = -x^2\cos x + \int 2x\cos x\,dx = -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$

$u = x^2 \quad dv = \sin x\,dx$

$du = 2x\,dx \quad v = -\cos x$

$u' = x \quad dv' = \cos x\,dx$

$du' = dx \quad v' = \sin x$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

(10) $\int x\ln x\,dx = \frac{x^2\ln x}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x}dx = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C$

$u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

(12) $\int \frac{\ln x}{\sqrt{x}}\,dx = 2\sqrt{x}\ln x - \int \frac{2\sqrt{x}}{x}dx = 2\sqrt{x}\ln x - 4\sqrt{x} + C$

$u = \ln x \quad dv = x^{-1/2}dx$

$du = \frac{1}{x}dx \quad v = 2\sqrt{x}$

(28) $\int \frac{xe^x}{(x+1)^2}\,dx = \frac{-xe^x}{x+1} + \int \frac{\cancel{(x+1)}}{\cancel{(x+1)}}e^x dx = \frac{-xe^x}{x+1} + e^x + C$

$= \frac{e^x}{x+1} + C$

$u = xe^x \quad dv = \frac{1}{(x+1)^2}dx$

$du = (x+1)e^x dx \quad v = \frac{-1}{x+1}$

(30) $\int_0^2 x e^{2x}\,dx = \frac{x}{2}e^{2x}\Big|_0^2 - \int_0^2 \frac{1}{2}e^{2x}dx = (e^{4x} - 0) - \frac{1}{4}e^{2x}\Big|_0^2$

$= e^{4x} - \frac{e^{4x}}{4} + \frac{1}{4} = \frac{3e^{4x}+1}{4}$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

(32) $\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x}\Big|_{\sqrt{e}}^{e} + \int_{\sqrt{e}}^{e} \frac{1}{x^2}\,dx = \left(\frac{-1}{e} - \frac{-1/2}{\sqrt{e}}\right) + \left(\frac{-1}{e} - \frac{-1}{\sqrt{e}}\right)$

$u = \ln x \quad dv = x^{-2}dx$

$du = \frac{1}{x}dx \quad v = \frac{-1}{x}$

$= \frac{-2}{e} + \frac{3}{2\sqrt{e}} = \frac{3\sqrt{e}-4}{2e}$

(41) (a) $\int e^{\sqrt{x}}\,dx = \int 2re^r\,dr = 2\left[re^r - \int e^r dr\right] = 2\left[re^r - e^r\right] = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

$r = \sqrt{x} \quad r^2 = x \qquad u = r \quad dv = e^r dr$

$2r\,dr = dx \qquad du = dr \quad v = e^r$

$= 2e^{\sqrt{x}}(\sqrt{x}-1) + C$

9.5

(10) $\int \frac{dx}{x^2+8x+7}$ Look at $\frac{1}{x^2+8x+7} = \frac{1}{(x+1)(x+7)} = \frac{A}{(x+1)} + \frac{B}{(x+7)}$

so $1 = A(x+7) + B(x+1)$

Let $x = -1$, then $1 = 6A \Rightarrow A = 1/6$

Let $x = -7$, then $1 = -6B \Rightarrow B = -1/6$

So $\int \frac{dx}{x^2+8x+7} = \int \frac{1}{6(x+1)}dx - \int \frac{1}{6(x+7)}dx = \frac{1}{6}\ln|x+1| - \frac{1}{6}\ln|x+7| + C$

$= \frac{1}{6}\ln\left|\frac{x+1}{x+7}\right| + C$

(12) $\int \frac{5x-5}{3x^2-8x-3}\,dx \Rightarrow 5x-5 = A(3x+1) + B(x-3) \Rightarrow A = 1$ and $B = 2$

(Let $x = 3$) (inspection after plugging in $A = 1$)

So $\int \frac{5x-5}{3x^2-8x-3}\,dx = \int \frac{1}{x-3}\,dx + \int \frac{2}{3x+1}\,dx = \ln|x-3| + \frac{2}{3}\ln|3x+1| + C$

14. $\int \frac{dx}{x(x^2-1)} \rightarrow \frac{1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \Rightarrow \underset{(\text{Let } x=0)}{A=-1},\ \underset{(\text{Let } x=-1)}{B=1/2},\ \underset{(\text{Let } x=1)}{C=1/2}$

$$1 = (x^2-1)A + x(x-1)B + x(x+1)C$$

$$\text{So } \int \frac{dx}{x(x^2-1)} = \int \frac{-1}{x}dx + \int \frac{1}{2(x+1)}dx + \int \frac{1}{2(x-1)}dx = -\ln|x| + \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C$$

$$= \frac{1}{2}\ln\left|\frac{x^2-1}{x^2}\right| + C = \frac{1}{2}\ln\left(\frac{|x^2-1|}{x^2}\right) + C$$

16. $\int \frac{x^2-4}{x-1}dx$

$$\begin{array}{r} x+1 \\ x-1 \overline{)\, x^2 + 0x - 4} \\ \underline{x^2 - x} \\ x - 4 \\ \underline{x - 1} \\ -3 \end{array}$$

$$\text{So } \int \frac{x^2-4}{x-1}dx = \int \left((x+1) - \frac{3}{x-1}\right)dx$$

$$= \int x\,dx + \int dx - 3\int \frac{1}{x-1}dx$$

$$= \frac{x^2}{2} + 1 - 3\ln|x-1| + C$$

18. $\int \frac{x^2}{x^2-3x+2}dx = \int 1 + \frac{3x-2}{x^2-3x+2}dx \qquad 3x-2 = A(x-2) + B(x-1)$

$$\Rightarrow A = -1,\ B = 4$$

$$= \int dx + \int \frac{-1}{x-1}dx + \int \frac{4}{x-2}dx = x - \ln|x-1| + 4\ln|x-2| + C$$

20. $\int \frac{2x^5 - x^3 - 1}{x^3 - 4x}dx = 2x^2 + 7 + \frac{28x-1}{x^3-4x}$ (by long division)

$$\frac{28x-1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} \Rightarrow A = \frac{1}{4},\ B = \frac{55}{8},\ C = \frac{-57}{8}$$

$$\text{So } \int \frac{2x^5-x^3-1}{x^3-4x}dx = \int 2x^2 + 7 + \frac{1}{4x} + \frac{58}{8(x-2)} - \frac{57}{8(x+2)}dx$$

$$= \frac{2x^3}{3} + 7x + \frac{1}{4}\ln|x| + \frac{58}{8}\ln|x-2| - \frac{57}{8}\ln|x+2| + C$$