**Section 9.5**

**Problem 28** We have

$$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

Multiplying through by $x(x^2+1)$, we see that

$$1 = A(x^2+1) + (Bx+C)x.$$

When $x = 0$, this becomes $1 = A$, and so $A = 1$. Plugging in $A = 1$, we find that

$$1 = x^2 + 1 + Bx^2 + Cx,$$

or

$$-x^2 = Bx^2 + Cx.$$

Thus $B = -1$ and $C = 0$. Therefore,

$$\int \frac{1}{x^3+x}\,dx = \int \left(\frac{1}{x} + \frac{-x}{x^2+1}\right)\,dx.$$

This integral can now easily be evaluated, since the integral distributes over the addition, and the first integral is $\ln|x|$, and the second can be evaluated by using the substitution $u = x^2+1$. In the end, we find that

$$\int \frac{1}{x^3+x}\,dx = \ln|x| - \frac{1}{2}\ln|x^2+1| + C.$$

**Problem 31** Long division shows that $x^2+1$ goes into $x^3-3x^2+2x-3$ a total of $x-3$ times with a remainder of $x$. Thus

$$\begin{aligned}\int \frac{x^3-3x^2+2x-3}{x^2+1}\,dx &= \int \left(x - 3 + \frac{x}{x^2+1}\right)\,dx \\ &= \frac{x^2}{2} - 3x + \frac{1}{2}\ln|x^2+1| + C,\end{aligned}$$

where the last part was found by way of the substitution $u = x^2+1$.