4. This problem can be done with either a u-substitution or Trig Substitution

(a) U-Substitution

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx \qquad u = x^2-1 \qquad x^2 = u+1$$
$$du = 2x\,dx \qquad dx = \frac{du}{2x}$$

$$= \frac{1}{2}\int \frac{u+1}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int \left(\sqrt{u} + \frac{1}{\sqrt{u}}\right)du$$

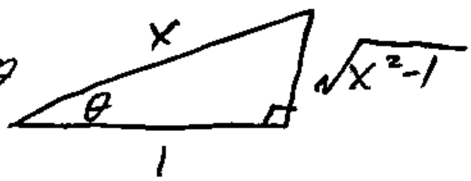
$$= \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + C\right)$$

$$= \frac{1}{2}u^{\frac{3}{2}} + u^{\frac{1}{2}} + C$$ Substitute the x's back in

$$= \boxed{\frac{1}{3}(x^2-1)^{\frac{3}{2}} + (x^2-1)^{\frac{1}{2}} + C}$$

(b) Trig Substitution

$$\int \frac{x^3}{\sqrt{x^2-1}}\,dx \qquad x = \sec\theta \qquad \sqrt{x^2-1} = \tan\theta$$
$$dx = \sec\theta\tan\theta\,d\theta$$



$$= \int \frac{(\sec\theta)^3(\sec\theta)(\tan\theta)\,d\theta}{\tan\theta}$$

$$= \int (\sec\theta)^4\,d\theta$$

$$= \int (\sec^2\theta)(\sec^2\theta)\,d\theta$$

$$= \int (1+\tan^2\theta)(\sec^2\theta)\,d\theta$$ We use a u-substitution

$$u = \tan\theta$$
$$du = \sec^2\theta\,d\theta$$

$$= \int (1+u^2)\,du$$

$$= u + \frac{u^3}{3} + C.$$

$u = \tan\theta = \sqrt{x^2-1}$, so we substitute this back in.

$$= \boxed{\frac{1}{3}(x^2-1)^{\frac{3}{2}} + (x^2-1)^{\frac{1}{2}} + C}$$