

7. (i) The integral is improper because it has an infinite limit of integration.

$$(ii) \int_0^{\infty} \frac{1}{e^x + e^{-x}}$$

$$u = e^x$$

$$du = e^x dx$$

$$dx = \frac{du}{u}$$

$$= \int_1^{\infty} \frac{1}{u + \frac{1}{u}} \frac{du}{u}$$

$$= \int_1^{\infty} \frac{1}{u^2 + 1} du$$

$$(iii) \int_1^{\infty} \frac{1}{u^2 + 1} du$$

$$= \arctan u \Big|_1^{\infty}$$

Substitute back in $u = e^x$. Make sure to change the limit of integration.

$$= \lim_{b \rightarrow \infty} \arctan e^x \Big|_0^b$$

$$= \lim_{b \rightarrow \infty} [\arctan e^b - \arctan 1]$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \boxed{\frac{\pi}{4}}$$