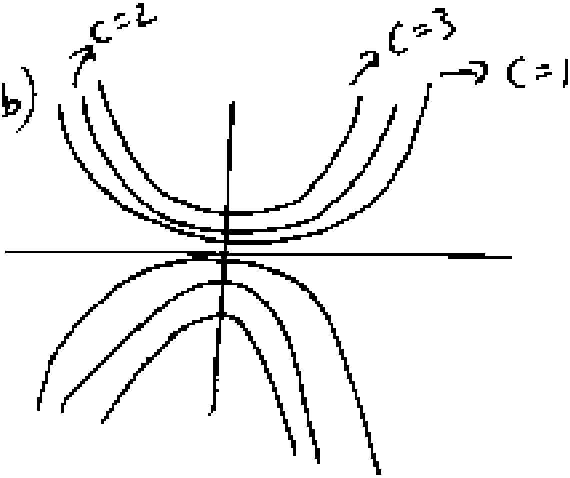
Section 9.1

6 a) $y = Ce^{x^2/2}$

$y' = Ce^{x^2/2}\left(\frac{2x}{2}\right) = xCe^{x^2/2} = xy$

b)



c) $y(0) = 5, \quad Ce^0 = 5, \quad C = 5$

thus, solution $\rightarrow$ $\boxed{y = 5e^{x^2/2}}$

d) $y(1) = 2, \quad Ce^{1/2} = 2, \quad C = 2e^{-1/2}$

thus, solution $\rightarrow$ $y = 2e^{-1/2}e^{x^2/2}$

$= 2e^{(x^2-1)/2}$

Section 9.1

② $y = \dfrac{2 + \ln x}{x}$

$y' = \dfrac{x\left(\frac{1}{x}\right) - (2 + \ln x)(1)}{x^2} = \dfrac{-1 - \ln x}{x^2}$ $+$ $y(1) = \dfrac{2 + \ln 1}{1} \stackrel{\checkmark}{=} 2$

SUB IN! SUB IN!

$x^2 y' + xy = x^2\left(\dfrac{-1 - \ln x}{x^2}\right) + x\left(\dfrac{2 + \ln x}{x}\right)$

$= (-1 - \ln x) + (2 + \ln x) \stackrel{\checkmark}{=} 1$

④ $y = e^{rt}$ $\quad$ $y'' + y' - 6y = 0$

$y' = re^{rt}$ $\quad$ $r^2 e^{rt} + re^{rt} - 6e^{rt} = 0$

$y'' = r^2 e^{rt}$

$(r^2 + r - 6)e^{rt} = 0$

FACTOR! $(r+3)(r-2) = 0$ $\quad$ $\boxed{r = -3, 2}$

Section 9.1

(8) a) if x is close to zero, slope must be nearly horizontal.

if x is large, slope must be nearly vertical.

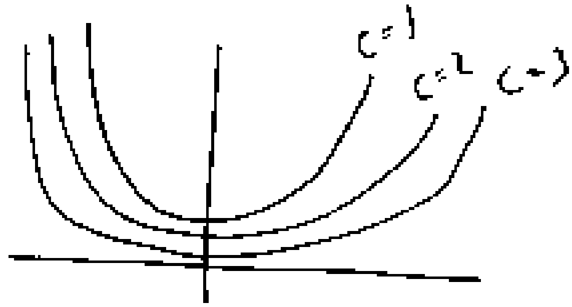
b) $y = (C - x^2)^{-1/2}$

$y' = x(C - x^2)^{-3/2}$

$xy^3 = x\left[(C-x^2)^{-1/2}\right]^3$

$= x(C - x^2)^{-3/2} = y'$

c)



d) $y(0) = (C-0)^{-1/2} = 1/\sqrt{C}$, $y(0) = 2$

$\sqrt{C} = \frac{1}{2}$, $C = \frac{1}{4}$

thus, $\boxed{y = \left(\frac{1}{4} - x^2\right)^{-1/2}}$

(10) a) $y = k$, $y' = 0$, thus, $\frac{dy}{dt} = y^4 - 6y^3 + 5y^2$

$0 = k^4 - 6k^3 + 5k^2$

$k^2(k^2 - 6k + 5) = 0$

$k^2(k-1)(k-5) = 0$

$k = 0, 1,$ or 5

b) y is increasing

c) y is decreasing

Section 9.1

(12) a) the coffee cools most quickly right when it's removed from the heat source.

rate → 0

coffee → room temp

b) $$\frac{dy}{dt} = k\,(y - R)$$

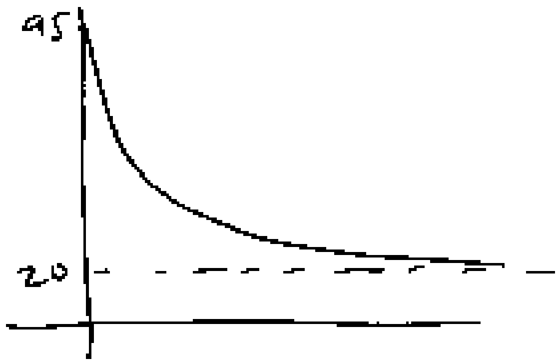
k → constant

y → temp. of coffee

R → room temp.

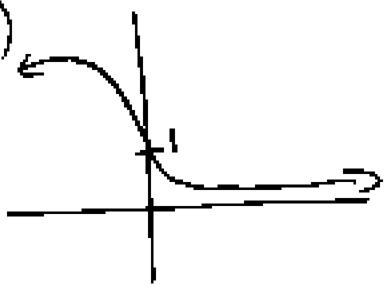
the model <u>is</u> <u>appropriate</u>!

c)

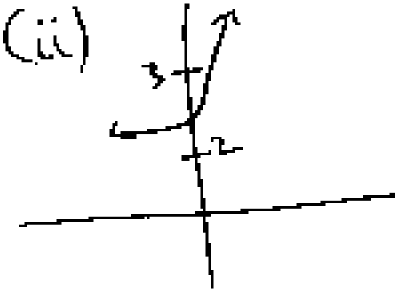


Section 9.2

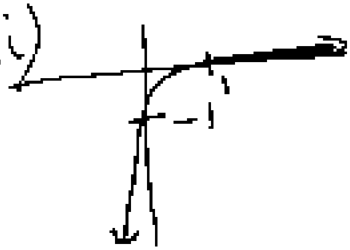
(2) a) (i)



(ii)



(iii)



b) for $c \le 2$, $\lim_{t \to \infty} y(t)$ is finite.

equilibrium sol. $\Rightarrow \boxed{y = 0,\ y = 2}$

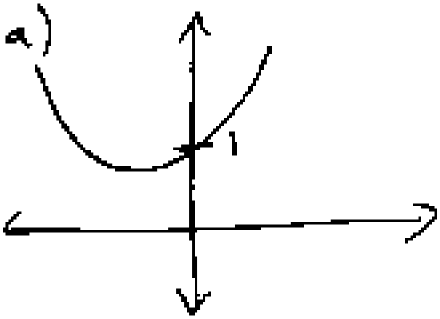
(3) $y' = y - 1$, $\boxed{\text{IV}}$ is the direction field

(4) $y' = y - x = 0$ on line $y = x$, $\boxed{\text{II}}$ is direction field

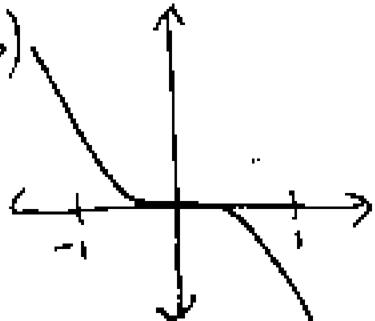
(5) $y' = y^2 - x^2 = 0$, $\boxed{\text{III}}$ is direction field

(6) $y' = y^3 - x^3 = 0$ on $y = x$, $\boxed{\text{I}}$ is direction field

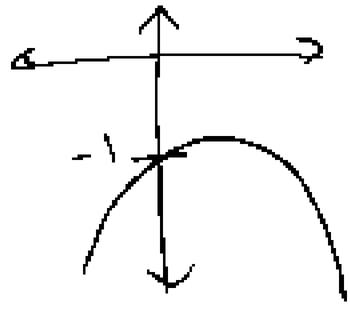
(8) a)



b)

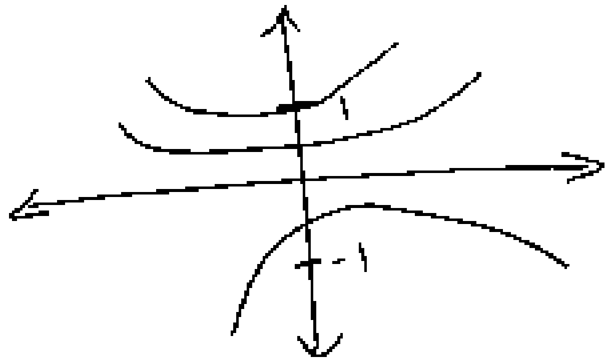


c)



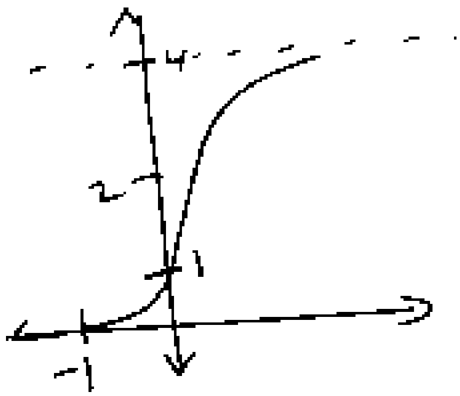
Section 9.2

(10) $y' = xy + y^2$



(14)

x	y	$y' = y(4-y)$
0	0	0
0	1	3
0	-1	-5
0	2	4
0	-2	-12
0	$\frac{1}{2}$	1.75
0	$-\frac{1}{2}$	-2.25
1	0	0
1	1	3
1	2	4
1	-1	-5



(24) a) $h = 0.2, \quad x_0 = 0, \quad y_0 = 1, \quad F(x,y) = 2xy^2$

$y_1 = 1 + 0.2(2 \cdot 0 \cdot 1^2) = 1, \quad y_2 = 1 + 0.2(2 \cdot 0.2 \cdot 1^2)$ $\boxed{= 1.08 \simeq y(0.4)}$

We need to find y_2 b/c $x_2 = 0.4$

b) $h = 0.1, \quad y_1 = 1 + 0.1(2 \cdot 0 \cdot 1^2) = 1, \quad y_2 = 1 + 0.1(2 \cdot .1 \cdot 1^2)$
$= 1.02$

$y_3 = 1.02 + 0.1(2 \cdot 0.2 \cdot 1.02^2) = 1.06162$

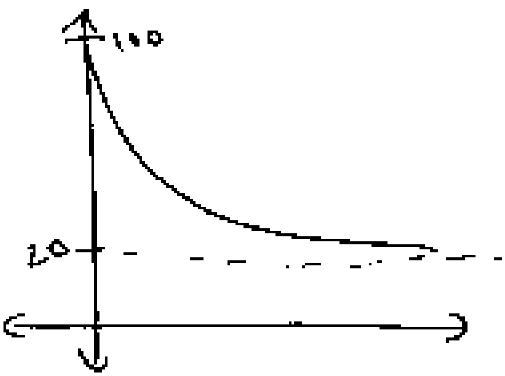
$y_4 = 1.06162 + 0.1(2 \cdot 0.3 \cdot 1.06162^2) = \boxed{1.1292 \simeq y(0.4)}$

(28) a) we have $\frac{dy}{dt} = k(y - R)$

$R = 20^\circ C$, $\frac{dy}{dt} = -1^\circ C/\text{min}$ when $y = 70^\circ C$

thus, $\frac{dy}{dt} = -\frac{1}{50}(y - 20)$

b)



c) $y_0 = 95$, $t_0 = 0$, $h = 2$

$y_1 = y_0 + hF(t_0, y_0) = 95 + 2\left[-\frac{1}{50}(95-20)\right] = 92$

$y_2 = y_1 + hF(t_1, y_1) = 92 + 2\left(-\frac{1}{50}(92-20)\right) = 89.1$

$y_3 = y_2 + hF(t_2, y_2) = 89.1 + 2\left[-\frac{1}{50}(89.1-20)\right] = 86.3$

$y_4 = y_3 + hF(t_3, y_3) = 86.3 + 2\left[-\frac{1}{50}(86.3-20)\right] = 83.7$

$y_5 = y_4 + hF(t_4, y_4) = 83.7 + 2\left[-\frac{1}{50}(83.7-20)\right] = 81.1$

Thus, $\boxed{y(10) = 81.1^\circ C}$

Section 9.3

(6) $y' = \frac{xy}{2\ln y}$

$$\int \frac{2\ln y}{y}\,dy = \int x\,dx$$

$$(\ln y)^2 = \frac{x^2}{2} + C$$

$$\ln y = \pm\sqrt{x^2/2 + C}$$

$$\boxed{y = e^{\pm\sqrt{\frac{x^2}{2} + C}}}$$

(12) $x + 2y\sqrt{x^2+1}\,\frac{dy}{dx} = 0 \qquad y(0) = 1$

$$x\,dx + 2y\sqrt{x^2+1}\,dy = 0$$

$$\int 2y\,dy = -\int \frac{x\,dx}{\sqrt{x^2+1}}$$

$$y^2 = -\sqrt{x^2+1} + C$$

$$1 = -1 + C \qquad \boxed{C = 2}$$

thus, $\boxed{y^2 = 2 - \sqrt{x^2+1}}$

Section 9.3

(16) $\dfrac{dy}{dx} = \dfrac{y^2}{x^3}$, $y(1) = 1$

$$\int \frac{dy}{y^2} = \int \frac{dx}{x^3}$$

$$-\frac{1}{y} = -\frac{1}{2x^2} + C$$

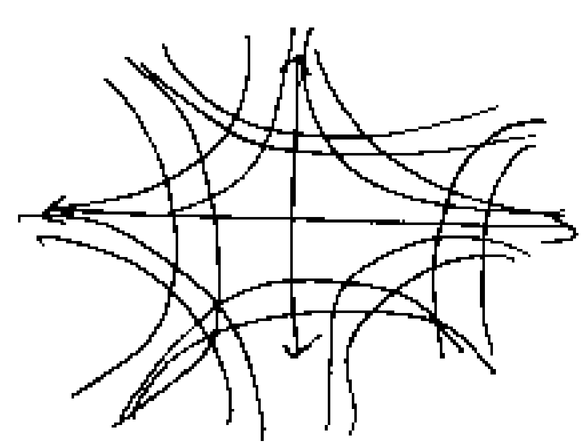
$$-1 = \frac{-1}{2} + C \quad \boxed{C = \frac{-1}{2}}$$

Thus, $\dfrac{1}{y} = \dfrac{1}{2x^2} + \dfrac{1}{2}$, $\boxed{y = \dfrac{2x^2}{x^2+1}}$

(24) The curves $x^2 - y^2 = k$ are hyperbolas.
The orthogonal trajectories must satisfy

$y' = -y/x$, $\dfrac{dy}{y} = \dfrac{-dx}{x}$, $\ln|y| = -\ln|x| + C$,

$\underline{xy = C}$



Section 9.3

(28) from 9.2.28: $\frac{dy}{dt} = \frac{-1}{50}(y-20)$

$$\int \frac{dy}{y-20} = \int \left(-\frac{1}{50}\right) dt$$

$$\ln(y-20) = \frac{-1}{50}t + C$$

$$y - 20 = Ke^{-t/50}$$

$$y(t) = Ke^{-t/50} + 20$$

$y(0) = 95$

$95 = K + 20$

$\boxed{K = 75}$

$\boxed{y(t) = 75e^{-t/50} + 20}$

(32) a) The amount of new currency introduced per day is $\frac{dx}{dt} = \frac{10-x}{10} \cdot 0.05$

$\boxed{= 0.005(10-x)}$ billion \$ per day

b) $\frac{dx}{10-x} = 0.005\, dt$

$$\int \frac{-dx}{10-x} = \int -0.005\, dt$$

$$\ln(10-x) = -0.005t + C$$

$$10 - x = Ce^{-0.005t}$$

$\downarrow$

$= e^{C}$

from $x(0) = 0$, $C = 10$

$\boxed{x(t) = 10(1 - e^{-0.005t})}$

c) \$9 billion $= 10(1 - e^{-0.005t})$

$t = 200(\ln 10)$

$\boxed{\begin{aligned} &= 460.5 \text{ days} \\ &= 1.26 \text{ years} \end{aligned}}$

Section 9.3

(34) a)

$$\frac{dy}{dt} = \left(0.05\,\frac{kg}{L}\right)\left(5\,\frac{L}{min}\right) + \left(.04\,\frac{kg}{L}\right)\left(10\,\frac{L}{min}\right) - \left(\frac{y(t)}{1000}\,\frac{kg}{L}\right)\left(15\,\frac{L}{min}\right)$$

$$\boxed{= \frac{130 - 3y}{200}\ \frac{kg}{min}}$$

now

$$\int \frac{dy}{130-3y} = \int \frac{dt}{200}$$

$\downarrow$ solve

$$130 - 3y = 130\, e^{-3t/200}$$

$$\boxed{y = \frac{130}{3}\left(1 - e^{-3t/200}\right) kg}$$

b) after an hour:

$$y = \frac{130}{3}\left(1 - e^{-180/200}\right) \boxed{= 25.7\ kg}$$