§ 11.1 : 8, (13), (14), 18, 24, 32, (42), 56, (62)

8. $a_1 = 1$

$a_{n+1} = \frac{1}{1+a_n}$ so sequence is: $\left\{1, \frac{1}{1+1}, \frac{1}{1+\frac{1}{2}}, \dots\right\}$

$seq = \left\{1, \frac{1}{2}, \frac{1}{\frac{3}{2}}, \dots\right\} = \left\{1, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \dots\right\}$

18. $a_n = \frac{\sqrt{n}}{1+\sqrt{n}} = \frac{1}{\frac{1}{\sqrt{n}}+1}$

so $a_n \to \frac{1}{0+1}$ as $n \to \infty$ $\therefore$ converges

24. $2n \to \infty$ as $n \to \infty$ so since $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$

$\lim_{n\to\infty} \arctan 2n = \pi/2$ $\therefore$ convergent

32. $a_n = \ln(n+1) - \ln n$
$= \ln\left(\frac{n+1}{n}\right)$
$= \ln\left(1 + \frac{1}{n}\right)$

$\lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right) = \ln(1) = 0$

convergent

56. $a_n = 3 + \frac{(-1)^n}{n}$ defines a sequence that is not monotonic.

seq: $\{2, 3.5, 2.\overline{6}, 3.25, 2.8, \dots\}$ not increasing
not decreasing

$2 \le a_n \le 3.5$ $\forall n \geq 1$ so
seq bounded

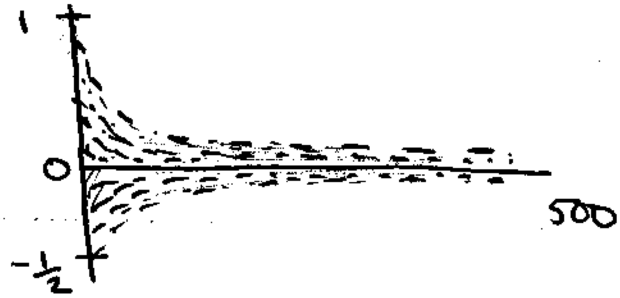
Optional Problems:

13. $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots\}$ each term $-\frac{2}{3}$ times previous term:

$a_n = \left(-\frac{2}{3}\right)^{n-1}$

14. $\{0, 2, 0, 2, 0, \ldots\}$ $\quad \frac{0+2}{2} = 1$: alternately subtract & add 1 to 1 to get seq.

$a_n = 1 - (-1)^{n-1}$

42. 

From graph: seq slowly $\to 0$

$$0 \le \frac{|\sin n|}{\sqrt{n}} \le \frac{1}{\sqrt{n}}$$

$$\to 0 \text{ as } n \to \infty$$

So by Squeeze Thm & Thm 5, $\left\{\frac{\sin n}{\sqrt{n}}\right\}$ converges to 0.

62. Let P_n be: $0 < a_{n+1} \le a_n \le 2$

$n=1$: $a_2 = \frac{1}{3-2} = 1$ ✓

Assume true for P_n. So $a_{n+1} \le a_n$

$\Rightarrow -a_{n+1} \ge -a_n \Rightarrow 3 - a_{n+1} \ge 3 - a_n$

$\Rightarrow a_{n+2} = \frac{1}{3-a_{n+1}} \le \frac{1}{3-a_n} = a_{n+1}$

Also $a_{n+2} > 0$ (b/c $3 - a_{n+1} > 0$) and

$a_{n+2} \le 2$ by induction hypothesis

So P_n true.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} \Rightarrow L = \frac{1}{3-L} \Rightarrow L^2 - 3L + 1 = 0 \Rightarrow L = \frac{3 \pm \sqrt{5}}{2}$

$L \le 2$ so $L = \frac{3-\sqrt{5}}{2}$