§7.3 : 4, 8, 10, 14, (17), 26, 30.

4.) $\displaystyle\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx.$ let $x = 4\cdot\sin\theta \Rightarrow dx = 4\cos\theta\cdot d\theta$

$$= \int_{\theta=0}^{\theta=\pi/3} \frac{64\cdot \sin^3\theta}{4\cdot\cos\theta}\cdot(4\cos\theta\, d\theta)$$

$$= \int_{\theta=0}^{\theta=\pi/3} 64\sin^3\theta\, d\theta$$

$$= 64\cdot\int_{\theta=0}^{\theta=\pi/3} \sin\theta(1-\cos^2\theta)\,d\theta$$

$$= 64\cdot\int_{\theta=0}^{\theta=\pi/3} (\sin\theta - \sin\theta\cdot\cos^2\theta)\,d\theta$$

$$= 64\cdot\left[-\cos\theta + 1/3\cdot\cos^3\theta\right]_{\theta=0}^{\theta=\pi/3}$$

$$= 64\cdot\left[(-1/2 + 1/24) - (-1+1/3)\right] = 64(5/24) = \underline{40/3}.$$

8) $\displaystyle\int \frac{x}{(x^2+4)^{5/2}}\,dx$ let $x = 2\tan\theta. \Rightarrow dx = 2\sec^2\theta\, d\theta.$

$$= \int \frac{2\tan\theta}{32\sec^5\theta}(2\sec^2\theta\, d\theta)$$

$$\int \frac{\tan\theta}{8\sec^3\theta}\,d\theta$$

$$= \frac{1}{8}\int \sin\theta\cos^2\theta\, d\theta$$

$$= 1/8\cdot(-1/3\cdot\cos^3\theta) + C$$

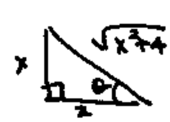
$$= -1/24\cdot\cos^3\theta + C.$$

$$= -\frac{1}{24}\cdot\left(\frac{2}{\sqrt{x^2+4}}\right)^3 + C$$

$$= -\frac{1}{3}\cdot\frac{1}{(x^2+4)^{3/2}} + \underline{C.}$$

$x/2 = \tan\theta.$

$\Rightarrow \cos\theta = 2/\sqrt{x^2+4}.$



OR let $u = x^2+4. \Rightarrow du = 2x\,dx.$

$$\int\frac{x\,dx}{(x^2+4)^{5/2}} = \int \tfrac{1}{2}u^{-5/2}\,du$$

$$= \tfrac{1}{2}\cdot\left(-\tfrac{2}{3}\right)\cdot u^{-3/2} + C$$

$$= -\tfrac{1}{3}\cdot u^{-3/2} + C$$

$$= -\tfrac{1}{3}\cdot(x^2+4)^{-3/2} + C.$$

10)

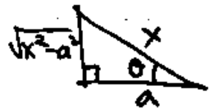
$$\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx.$$ Let $x = a\sec\theta \Rightarrow dx = a\cdot\sec\theta\tan\theta\cdot d\theta$

$$= \int \frac{a\tan\theta}{a^4\sec^4\theta}\cdot(a\sec\theta\tan\theta)\,d\theta$$

$$\frac{1}{a^2}\int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$

$$\frac{1}{a^2}\int \sin^2\theta\cos\theta\cdot d\theta$$

$$1/a^2\cdot\left(1/3\cdot\sin^3\theta\right)+C$$ $x/a=\sec\theta$



$$= \frac{1}{3a^2}\cdot\left(\frac{\sqrt{x^2-a^2}}{x}\right)^3+C.$$

$$\frac{(x^2-a^2)^{3/2}}{3a^2\cdot x^3}+C.$$

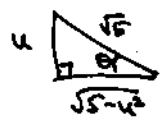
14)

$$\int \frac{du}{u\sqrt{5-u^2}}.$$ Let $u = \sqrt{5}\cdot\sin\theta \rightarrow du = \sqrt{5}\cdot\cos\theta\,d\theta.$

$$= \int \frac{1}{(\sqrt{5}\cdot\sin\theta)(\sqrt{5}\cdot\cos\theta)}\cdot(\sqrt{5}\cdot\cos\theta\,d\theta)$$

$$= \frac{1}{\sqrt{5}}\int \csc\theta\,d\theta.$$

$$= -1/\sqrt{5}\cdot\ln|\csc\theta+\cot\theta|+C.$$ $\sin\theta = u/\sqrt{5}.$



$$= -\frac{1}{\sqrt{5}}\cdot\ln\left|\frac{\sqrt{5}+\sqrt{5-u^2}}{u}\right|+C.$$

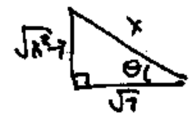
17)

1) $\int \frac{x}{\sqrt{x^2-7}}\,dx$ let $x=\sqrt{7}\cdot\sec\theta \Rightarrow dx=\sqrt{7}\cdot\sec\theta\tan\theta\,d\theta$

$= \int \frac{\sqrt{7}\cdot\sec\theta}{\sqrt{7}\cdot\tan\theta}\cdot(\sqrt{7}\sec\theta\tan\theta)\,d\theta$

$= \sqrt{7}\int \sec^2\theta\,d\theta$

$= \sqrt{7}\cdot\tan\theta + C$ $x/\sqrt{7}=\sec\theta.$



$= \sqrt{7}\left(\sqrt{x^2-7}/\sqrt{7}\right)+C.$

$= \sqrt{x^2-7}+C$

OR: let $u=x^2-7. \Rightarrow du=2x\,dx.$

$\int \frac{1}{2}\cdot u^{-1/2}\,du$

$= \frac{1}{2}\cdot 2u^{1/2}+C$

$= u^{1/2}+C = \sqrt{x^2-7}+C.$

26)

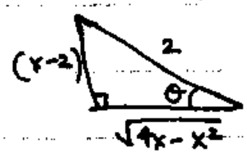
1) $\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

$= \int \frac{x^2}{\sqrt{4-(x-2)^2}}\,dx$ let $x=2+2\sin\theta \Rightarrow dx=2\cos\theta\,d\theta.$

$= \int \frac{(4+8\sin\theta+4\sin^2\theta)}{2\cos\theta}\cdot(2\cos\theta\,d\theta)$

$= 4\int(1+2\sin\theta+\sin^2\theta)\,d\theta.$

$= 4\left(\theta - 2\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)+C$ $\sin\theta = \frac{x-2}{2}.$



$= 6\theta - 8\cos\theta - \sin 2\theta + C$

$= 6\cdot\sin^{-1}\left(\frac{x-2}{2}\right) - 8\left(\frac{\sqrt{4x-x^2}}{2}\right) - 2\left(\frac{x-2}{2}\right)\left(\frac{\sqrt{4x-x^2}}{2}\right)+C$

$= 6\cdot\sin^{-1}\left(\frac{x-2}{2}\right) - 4\sqrt{4x-x^2} - \frac{1}{2}(x-2)\left(\sqrt{4x-x^2}\right)+C$

30)

$$\int \sqrt{e^{2t}-9}\,dt. \quad \text{Let } t = \ln(3\sec\theta) \Rightarrow dt = \tan\theta\,d\theta$$

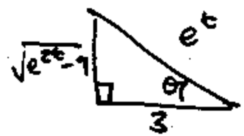
$$= \int 3\tan\theta \cdot (\tan\theta\,d\theta)$$

$$= 3\int \tan^2\theta\,d\theta.$$

$$= 3\cdot\int (\sec^2\theta - 1)\,d\theta$$

$$= 3\cdot(\tan\theta - \theta) + C$$

$\sec\theta = e^t/3.$



$$3\left(\sqrt{e^{2t}-9}/3 - \cos^{-1}(3/e^t)\right) + C$$

$$= \sqrt{e^{2t}-9} - 3\cos^{-1}(3e^{-t}) + C.$$