5.(i) $\frac{dy}{dx} = \frac{y}{1+\cos x}$

$$\int \frac{dy}{y} = \int \frac{dx}{1+\cos x}$$

$$\ln|y| = \int \frac{dx}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}$$ We multiply the numerator and denominator by $1-\cos x$

$$= \int \frac{1-\cos x}{1-\cos^2 x} dx$$

$$= \int \frac{1-\cos x}{\sin^2 x} dx$$

$$= \int \csc^2 x - \int \frac{\cos x}{\sin^2 x} dx$$ Let $u = \sin x$
$du = \cos x\, dx$

$$\ldots - \int \frac{du}{u^2}$$

$$- \int u^{-2} du$$

$$+ \frac{1}{u}$$ Substitute $u = \sin x$

$$\ln|y| = -\cot x + \frac{1}{\sin x} + C$$

$$y = e^{-\cot x + \frac{1}{\sin x} + C}$$

(ii) We substitute $x = \frac{\pi}{2}$ and $y = 1$

$$1 = e^{-0+1+C}$$ Since $e^0 = 1$, $0+1+C=0$
$C = -1$

The particular solution is

$$y = e^{-\cot x + \frac{1}{\sin x} - 1}$$