4) $\sum_{n=1}^{\infty} \frac{1000}{\sqrt{n}+n^{1/3}}$ converges if and only if $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+n^{1/3}}$ does.

Use Limit Comparison with $\sum \frac{1}{\sqrt{n}}$.

$$L = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}+n^{1/3}}} = \lim_{n\to\infty} \frac{\sqrt{n}+n^{1/3}}{\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1+n^{1/3-1/2}}{1}$$

$$= \lim_{n\to\infty} 1 + 1/n^{1/6} = 1 \begin{smallmatrix} >0 \\ <\infty \end{smallmatrix}.$$ So, since $\sum \frac{1}{\sqrt{n}}$ is p-series with $p=\frac{1}{2}<1$ it diverges, and so does original series

$\boxed{\text{Diverges}}$

5) Notice that $\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0, & n \text{ is even} \\ 1, & n \text{ is } 1, 5, 9, \ldots \\ -1, & n \text{ is } 3, 7, 11, \ldots \end{cases}$

So $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{2}\right)}{\sqrt{n}} = \sum_{k=1}^{\infty} \left[\frac{(-1)^{k+1}}{\sqrt{2k-1}} + \frac{0}{\sqrt{2k}}\right]$ ← you don't have to write this out in summation form.

Now, $\sum \frac{0}{\sqrt{2k}} = \sum 0$ clearly converges.

$\sum \frac{(-1)^{k+1}}{\sqrt{2k-1}}$. Use Alt. series test.

a) $\lim_{k\to\infty} \frac{1}{\sqrt{2k-1}} = 0$

b) $\frac{1}{\sqrt{2k-1}}$ is clearly decreasing

So converges.

Our original series converges since these two do.

Check if absolutely convergent: $\sum_{n=1}^{\infty} \left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\sqrt{n}}\right| = \sum_{k=1}^{\infty} \frac{1}{\sqrt{2k-1}}$

Using Limit Comparison to $\sum \frac{1}{\sqrt{k}}$ (p-series with $p=\frac{1}{2}<1$), which diverges, we see that $\sum_{n=1}^{\infty} \left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\sqrt{n}}\right|$ diverges.

So $\boxed{\text{Cond. Conv.}}$