Additional Problems for Mathematics 1b
Handout B

If f is a function, then $P_n(x)$, the Taylor polynomial about $x = 0$ associated to f, is to be thought of as a "good" approximation of f for x near zero. Let's consider the polynomial

$$f(x) = x^3 - 4x^2 + 4x.$$

(a) What do you think the best linear approximation of this polynomial (approximation of the form $P_1(x) = a_0 + a_1 x$) ought to be at the point $b = 0$? Why?

What is the best quadratic approximation of the polynomial f (approximation of the form $P_2(x) = a_0 + a_1 x + a_2 x^2$) at the point $b = 0$?

What is the best cubic approximation of the polynomial f (approximation of the form $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$) at the point $b = 0$? Does this surprise you? Why or why not?

(b) The number 2 is a critical point of f. If we write $P_n(x) = a_0 + a_1(x-2) + a_2(x-2)^2 + ...a_n(x-2)^n$ then a_1 will be zero. Why?

In addition, a_4, a_5, ..., a_n will all be zero. Why is that?

(c) If b is any real number and we write $P_n(x) = a_0 + a_1(x-b) + a_2(x-b)^2 + ... + a_n(x-b)^n$ then what can we say about a_4, a_5 .....? Why?