Handout F

1. A chocolate truffle is a wonderfully decadent chocolate concoction. Truffles tend to be spherical or hemispherical.

   (a) Consider a truffle made by dipping a round hazelnut into various chocolates, building up a delicious chocolate delicacy. The number of calories per cubic millimeter varies with x, the distance from the center of the hazelnut. If $\rho(x)$ gives the calories per cubic millimeter at a distance x millimeters from the center, write an integral that gives the number of calories in a truffle of radius R.

   (b) Another truffle is made in a hemispherical mold of radius R. (The mold looks like a tiny hemispherical bowl.) Different layers of chocolate are poured into the mold, one at a time, and allowed to set. The number of calories per cubic millimeter varies with x, the distance from the top of the mold. The caloric density is given by $\delta(x)$ calories per cubic millimeter. Write an integral that gives the number of calories in this hemispherical truffle.

2. Suppose that $y = f(x)$ is a solution to the differential equation

   $$y'' + y' = -x^2.$$

   Why can't the the graph of $f(x)$ ever be both increasing and concave up?

3. Let f be a continuous function on $[a, b]$. We are interested in comparing the average value, f_{ave}, of f to the value $f((a+b)/2)$ of f at the midpoint of the interval.

   (a) Assume $f''(x) = 0$ for all x on $[a, b]$. Show that $f_{\text{ave}} = f((a+b)/2)$. [Hint: what kind of function has second-derivative always equal to zero?]

   (b) Assume $f''(x) > 0$ for all x on $[a, b]$, i.e. f is concave up. Show that $f_{\text{ave}} > f((a+b)/2)$. Draw a picture to illustrate your reasoning.

   (c) Assume $f''(x) < 0$ for all x on $[a, b]$, i.e. f is concave down. Show that $f_{\text{ave}} < f((a+b)/2)$. Draw a picture to illustrate your reasoning.