Exam #1 Review Worksheet Part I

Math 1B
October '99

Topics covered:
- Series (1) Definition of Convergence
- Examples (2) Geometric Series
  - (3) P-Series
  - (4) Alternating Series
- Tests for Convergence
  - (5) - Divergence Test
  - (6) - Comparison Tests
  - (7) - Ratio Test (for absolute convergence)

in Review Part II { - Power Series
- Taylor/Maclaurin Series

(1) Series: $a_1 + a_2 + a_3 + \cdots \quad = \sum_{k=1}^{\infty} a_k$

converges if

Ex. 1) Does $3 - 3 + 3 - 3 + \cdots$ converge?

(2) Geometric Series

general form $a + ar + ar^2 + \cdots = \sum_{k=1}^{\infty} ar^k$

- converges to $\frac{a}{1-r}$ when
- diverges when

Example 2) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \cdots = \sum_{k=0}^{\infty} \frac{1}{3 \cdot 2^k}$

Example 3) $\sum_{k=3}^{\infty} 2 \cdot \left(\frac{1}{2}\right)^k$

Review continued

Note. Can alway add take away a finite number of terms from a series without changing whether or not it converges!

Example 4 $+3+5+4+2+ \quad + \frac{1}{2} + {}_4 +$

Which of the following converge?

Example 5 $\sum_{k=1} \frac{3^{2k}}{2^{3k}}$

Example 6 $\sum_{k=1}^{\infty} \frac{2^{3k}}{3^{2k}}$

(3) p-series

general form

converges when

diverges fo

Example 7 $+\frac{1}{8}+\frac{1}{27}+ \qquad \sum_{k=1} \frac{}{k^3}$

Example 8 for what values of r does $\sum_{k=10}^{\infty} \frac{}{k^{3r}}$ converge?

(4) Alternating series

general form

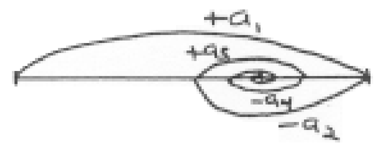
Example 9 $\frac{1}{2}+\frac{1}{3} \quad \frac{1}{4}+ \qquad \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$

Example 10 $0.1 \quad 0.11 + 0.111 \quad 0.1111 +$

Review continued

Alternating Series Test

Remember the visual mnemonic:



Accuracy of partial sums:

$\Rightarrow$ So if have a converging alternating series $a_1 - a_2 + a_3 - \cdots$ with sum S, then

<u>Example 11</u> Does $\sum_{k=1}^{\infty} \frac{(-1)^k k}{k+1}$ converge?

<u>Example 12</u> Does $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3 + \sqrt{k}}$ converge?

(5) Tests for Convergence – Divergence Test

<u>Example 13</u> for what values of r does $\sum_{k=1}^{\infty} (k+1) r^2$ converge

<u>Example 14</u> does $\sum_{k=1}^{\infty} \frac{k}{3k-5}$ converge?

<u>Example 15</u> does $\sum_{z=10}^{\infty} \frac{z^2(z+3)}{z}$ converge?

Review Continued

(6) Comparison Tests

Given two series $\sum_{k=1} a_k$ and $\sum^{\infty} b_k$ with a_k b_k nonnegative and $a_k \leq b_k$ for all k then

Example 16 Does $3 + \frac{1}{6} \quad 11 + \frac{1}{18} + \quad \sum_{k=1}^{\infty} \quad k^2+2$ converge?

Example 17 Does $\sum_{w=1}^{\infty} \frac{w+}{w^3}$ converge?

Don't forget you've also got the Limit Comparison Test

Example 18 $\sum_{t=1}^{\infty} \frac{1}{(t-3.1)^2}$ does this converge?

(7) Ratio Test (for Absolute Convergence)

based same concept as geometr. series

$1+r \quad r^2+$ converging when $|r|<1$

diverging otherwise

Given a series $\sum_{k=1}^{\infty} a_k$

Review continued

<u>Example 19</u> Does $\sum_{k=1}^{\infty} \frac{k+1}{2 \cdot k!}$ converge?

<u>Example 20</u> For what values of u does $\sum_{s=1}^{\infty} \frac{\pi \cdot u^s}{s!}$ converge?

<u>Example 21</u> Does $\sum_{z=1}^{\infty} \frac{1}{(.8^z) z}$ converge?

Answers to Review Session Examples

Example 1 nope, partial sums bounce between 3 and 0, so limit doesn't exist.

Example 2 $\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\cdots$ first term $=\frac{1}{3}$, ratio $=\frac{1}{2}$ so sum $=\frac{1/3}{1-1/2}=\frac{2}{3}$

Example 3 first term is $2\cdot\frac{1}{8}=\frac{1}{4}$, ratio $\frac{1}{2}$
($k=3$ starts) sum $=\frac{1/4}{1-1/2}=2/4=1/2$

Example 4 $=9+4+2+1+\cdots=9+$ geom. series w/ first term 4, ratio $1/2$, sum $=\frac{4}{1-1/2}=8$

sum $=9+8=17$

Example 5 $=\sum_{k=1}^{\infty}\frac{9^k}{8^k}=\sum_{k=1}^{\infty}\left(\frac{9}{8}\right)^k$ geom. series with ratio $9/8$ $\Rightarrow$ diverges

Example 6 $=\sum_{k=1}^{\infty}\frac{8^k}{9^k}=\sum_{k=1}^{\infty}\left(\frac{8}{9}\right)^k$ geom. series 1^{st} term $=8/9$, ratio $=8/9$, sum $=\frac{8/9}{1-8/9}=8$

Example 7 converges, p-series with $p=3$

Example 8 $\sum_{k=1}^{\infty}\frac{1}{k^{3r}}$ is a p-series with $p=3r$, converges if $p=3r>1$, or when $r>1/3$ (and diverges for $r\le\frac{1}{3}$)

Example 9 Alternating harmonic series $\Rightarrow$ converges

Example 10 does not converge, terms are increasing in absolute value

Example 11 $a_k=\frac{k}{k+1}$, $\lim_{k\to\infty}a_k=1$, fails alternating series test
also $a_1<a_2<a_3<\cdots$
diverges

Example 12 converges $\lim_{k\to\infty}\frac{1}{k^3+\sqrt{k}}=0$, and $a_1=\frac{1}{2}>a_2=\frac{1}{8+\sqrt{2}}>a_3$ etc.

Answers to Review Examples continued

Example 13 only when $r=0$! otherwise $\lim_{k\to\infty} (k+1)r^k \neq 0$

Example 14 $\lim_{k\to\infty} \frac{k}{3k-5} = \frac{1}{3}$, and so divergence test $\Rightarrow$ diverges

Example 15 $\lim_{z\to\infty} \frac{z^2(z+3)}{z^3-1} = \lim_{z\to\infty} \frac{z^3+3z^2}{z^3-1} = \lim_{z\to\infty} \frac{1+3/z}{1-1/z^3} = 1$, series diverges

Example 16 converges, compare to $\sum_{k=1}^{\infty} \frac{1}{k^2}$ (converges, p-series with $p=2>1$)

Example 17 converges $= \sum_{w=1}^{\infty} \frac{1+1/w}{w^2}$ and $\frac{1+1/w}{w^2} \leq \frac{2}{w^2}$ (converges, p-series, $p=2>1$)

Example 18 compare once again with the p-series $\sum_{t=1}^{\infty} \frac{1}{t^2}$, which converges $\lim_{t\to\infty} \left(\frac{1}{(t-3.1)^2} \Big/ \frac{1}{t^2}\right)$

$$= \lim_{t\to\infty} \left(\frac{t^2}{(t-3.1)^2}\right) = 1$$, so original series in question also converge.

Example 19 $a_k = \frac{k+1}{2\cdot k!}$ $\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left|\frac{k+1+1}{2\cdot(k+1)!} \Big/ \frac{(k+1)}{2k!}\right|$

$$= \lim_{k\to\infty} \left(\left(\frac{k+2}{k+1}\right)\cdot\frac{1}{k+1}\right) = 0 \Rightarrow \text{converges}$$

Example 20 $\lim_{s\to\infty} \left|\frac{\pi\cdot u^{s+1}}{(s+1)!} \Big/ \frac{\pi\cdot u^s}{s!}\right| = \lim_{s\to\infty} \left|\frac{u}{s+1}\right| = 0$ for all u, so converges for all u

Example 21 $\lim_{z\to\infty} \left|\frac{1}{.8^{z+1}\cdot(z+1)} \Big/ \frac{1}{.8^z\cdot z}\right| = \lim_{z\to\infty} \left(\frac{1}{.8}\left(\frac{z}{z+1}\right)\right) = \frac{1}{.8} = 1.25 > 1$ diverges