Problem Sets for October

Problem Set # 7
Read the series handout.

In the handout do problems on p.931 #1, 2, 4, 9, 12, 13 plus the following:

Problem:

If f is a function, then $P_n(x)$, the Taylor polynomial about $x = 0$ associated to f is to be thought of as a "good" approximation of f for x near zero. Let's consider the polynomial

$$f(x) = x^3 - 4x^2 + 4x.$$

(a) What do you think the best linear approximation of this polynomial (approximation of the form $P_1(x) = a_0 + a_1 x$) ought to be at the point $b = 0$? Why?

What is the best quadratic approximation of the polynomial f (approximation of the form $P_2(x) = a_0 + a_1 x + a_2 x^2$) at the point $b = 0$?

What is the best cubic approximation of the polynomial f (approximation of the form $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$) at the point $b = 0$? Does this suprise you? Why or why not?

(b) The number 2 is a critical point of f. If we write $P_n(x) = a_0 + a_1(x-2) + a_2(x-2)^2 + ...a_n(x-2)^n$ then a_1 will be zero. Why?

In addition, a_4, a_5, ..., a_n will all be zero. Why is that?

(c) If b is any real number and we write $P_n(x) = a_0 + a_1(x-b) + a_2(x-b)^2 + ... + a_n(x-b)^n$ then what can we say about a_4, a_5 .....? Why?

Problem Set # 8
Read §8.7
Do §8.7 # 12, 14, 18, 20, 23, 27, 29, 36

Problem Set # 9
Read §8.8
Do §8.7 # 25, 34, 38 and §8.8 # 1, 4, 9, 16a (b and c are extra credit)

Problem Set 10
Read §8.9 and do §8.9 # 3, 19, 20, 22, p. 641 # 40
plus:

1. In this problem you'll compare the error analysis arrived at using the alternating series error estimate with that gotten using the Taylor Remainder.

   Let $f(x) = x^{1/3}$.

   (a) Find the third order Taylor Polynomial, $T_3(x)$, for f at $x = 27$.
   (b) Use $T_3(x)$ found in part (a) to approximate $28^{1/3}$.
   (c) Find an upper bound for the error in this approximation by using the alternating series error estimate.
   (d) Now find an upper bound for the error in this approximation by using the Taylor Remainder (i.e., Taylor's Inequality).

2. The interval of convergence of the Maclaurin series for $\ln(1+u)$ is $u \in (-1, 1]$. On this interval the series converges to $\ln(1+u)$.

   (a) Using any method you like, find the Maclaurin series for $\ln(1+u)$.
   (b) By setting $u = x - 1$ in part (a), find a power series expansion for $\ln x$ centered at $x = 1$.
   (c) Find the Taylor series for $\ln x$ at $x = 1$ by taking derivatives. Make sure your answers to parts (b) and (c) agree. (They ought to because *if* a function has a power series expansion in $(x-1)$ then that expansion will be the Taylor series about $x = 1$.)

Tuesday Thursday classes: Let's assign the problems from PS 10 (Wednesday's assignment) on Tuesday and those from PS 11 on Thursday.

Problem Set 11

§8.9 # 23, 25 (See notes below; they are meant to guide you through the problem.)

In #25 we are looking at the situation for which d is much much smaller than D, so $\frac{d}{D}$ is small. We would like to use only the first few terms of a Taylor series, so we need to write a series in u where u is very small. That's why the book suggests writing a series in powers of $u = \frac{d}{D}$. Begin by manipulating the expression given until you have

$$\frac{q}{D^2} - \frac{q}{D^2}\left[\frac{1}{\left(1+\frac{d}{D}\right)^2}\right] = \frac{q}{D^2}\left[1 - \frac{1}{\left(1+\frac{d}{D}\right)^2}\right].$$

You'll expand the expression $\frac{1}{\left(1+\frac{d}{D}\right)^2}$. This looks like $\frac{1}{(1+u)^2}$ where you can think of $\frac{d}{D}$ as u. You'll only use the first two terms of the series to get the desired result.

As a start for your exam review do the Concept Check: p. 640. # 3, 4. 10, 11a-d and, on the same page, the true/false quiz #1-7, 11 plus the question below.

(I.) Amanda asked her friend Charlie for some help when studying for a math test on series. Charlie had quick and easy methods, but he sometimes said things that are incorrect. Your job in this problem is to pay very careful attention to Amanda and Charlie's conversation and correct Charlie wherever necessary.

1. Amanda asked Charlie how you can tell if the series, $1 + (-1) + (-1)^2 + (-1)^3 + (-1)^4 + \ldots$ converges or diverges. Charlie answered, "That's easy. This is just a geometric series with $a = 1$ and $r = -1$. So you just plug into the formula, $\frac{a}{1-r} = \frac{1}{1-(-1)} = \frac{1}{2}$. So, it converges to one half." Do you agree with Charlie's statement? Explain why or why not.

2. Amanda told Charlie that she was having a lot of trouble with geometric series. She asked Charlie how you could find out whether an expression like: $3 + 3\cdot 7^2 + 3\cdot 7^4 + 3\cdot 7^6 + \ldots + 3\cdot 7^{20}$ converged or not, and if it did, what it converged to. Charlie answered, "Okay, these are two step problems. First, that's a geometric series with $a = 3$, $r = 7^2$. So, to get the total, you just plug into the formula: $\frac{a(1-r^{n+1})}{1-r} = \frac{3(1-49^{21})}{1-49}$." Charlie told Amanda that you can work that out on a calculator. (Is Charlie's closed form correct? Explain below.)

   "You can tell the series doesn't converge" Charlie continued," because the r is 49. That's greater than one, so the series doesn't converge." Is Charlie's statement about convergence accurate? Why or why not?

3. The last question that Amanda asked was about a general infinite series like $\sum_{k=1}^{\infty} a_k$. She said that she remembered hearing something about looking at the value of a_k when k gets really, really big, and that can tell you something about whether the series $\sum_{k=1}^{\infty} a_k$ converges or not. Charlie responded, "Oh yeah, this makes convergence and divergence really easy. If you look at what the formula for a_k is, and if that formula gets closer and closer to zero then the series converges. Otherwise, the series diverges." Do you think Charlie is gave Amanda very solid advice? What advice would you have given her?

[NOTE: We will have the solutions to both problem set #10 and 11 up on the web by Saturday. This way students can check and correct their answers while studying for the exam. This problem set will not have to be handed in for a grade, but should be done. Students will be held responsible for the material on it!]

Problem Set 12

Study for the exam. There is nothing to hand in -(but let's keep the number 12 for the empty problem set.)

Problem Set 13
§5.5 # 4, 5, 8, 10, 14, 16, 20, 24, 27, 28, 32.

Looking at Integration from a Graphical Perspective due Wed, 10/24 or Tuesday, 10/23

This worksheet encourages you to make estimations, use symmetry, and, in general, take a graphical look at definite integrals. You are welcome to do the worksheet on your own – but if you'd like to work as a group, the Course Assistants will facilitate this during their problem sessions this week.

For each of the following, use an appropriate graph to evaluate the truth of falsehood of each claim. (Graphing calculators or computers can produce graphs for you.) Your answers should include pictures.

- Claim 1. $0 < \int_0^a e^{-x^2}\,dx < a$
- Claim 2. $\int_0^{\sqrt{\pi}} \cos(x^2)\,dx < 0$
- Claim 3. $\int_{-\pi}^{\pi} e^{-x^2/\sqrt{2}}\,dx = 2\int_0^{\pi} e^{-x^2/\sqrt{2}}\,dx$
- Claim 4. $\int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx < \int_1^2 \frac{1}{\sqrt{1+x^4}}\,dx$
- Claim 5. $\int_{-3}^{3} \frac{x}{1+x^4}\,dx > 0.001$
- Claim 6. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is greater than $2ab$ and less than $4ab$.

  plus:

  On the time interval $[a, b]$ a car's velocity, $v(t)$, is positive and increasing. The velocity is increasing at a decreasing rate on this interval. Suppose we partition the interval $[a, b]$ into 10 equal subintervals, each of length Δt. Let $t_k = a + k\Delta t$ where $k = 0, 1, 2, \cdots 10$.
- Claim 7. $\sum_{k=1}^{10} v(t_{k-1})\Delta t >$ the distance travelled on $[a, b]$.
- Claim 8. $\sum_{k=1}^{10} v(t_k)\Delta t >$the distance travelled on $[a, b]$.
- Claim 8. $\frac{1}{2}\left[\sum_{k=1}^{10} v(t_k)\Delta t + \sum_{k=1}^{10} v(t_{k-1})\Delta t\right] <$the distance travelled on $[a, b]$.

Problem Set 14
§5.5 # 57, 60 §5.6 # 4, 5, 8, 14, 22, 28 plus p. 438 # 5 (as review from first semester calculus)
Extra credit #34 in §5.6