Solution Set #21

Problems: 6.2 3, a w/ y=4, y=5, x=-1, 23, 31, 35, 46

Extra: 48

3) $y=x^2 \quad 0\le x\le 2 \quad y=4 \quad x=0$

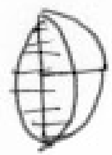
a) Disk with radius $\frac{1}{x}$ $\quad A=\pi\left(\frac{1}{x}\right)^2$

$$\int_0^4 A(y)\,dy = \int_0^4 \pi\sqrt{y}^{\,2}\,dy = \pi\int_0^4 y\,dy = \pi\left[\tfrac{1}{2}y^2\right]_0^4 = \boxed{8\pi}$$

b) $y=4$

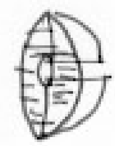
$A(x) = \pi(4-x^2)^2$

$$\int_0^2 \pi(4-x^2)^2\,dx$$



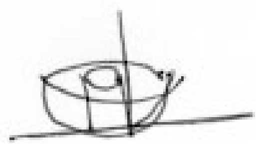
c) $y=5$

$$\pi\int_0^2\left[(5-x^2)^2 - 1\right]dx$$



d) $x=-1$

$$\int_0^4\left[(\sqrt{y}+1)^2-1\right]dy$$



10 a) $V=\int_1^3 \pi\left\{\left[\frac{1}{x}-(-1)\right]^2-\left[0-(-1)^2\right]\right\}dx = \pi\int_1^3\left[\left(\frac{1}{x}+1\right)^2-1^2\right]dx$

$$= \pi\int_1^3 \frac{1}{x^2}+\frac{2}{x} = \pi\left[-\frac{1}{x}+2\ln x\right]_1^3 = \boxed{2\pi\left(\ln 3+\tfrac{1}{3}\right)}$$

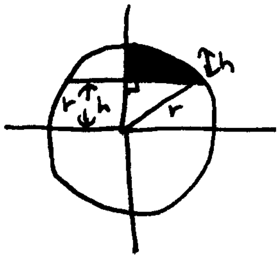
b) $y=4 \quad x=1 \quad y=\frac{1}{x} \quad y=1 \quad x=1 \quad x=3 \quad y=\frac{1}{3}$

$$\int_1^3 \pi\left(4-\frac{1}{x}\right)^2 dx$$

c) $\int_1^3 \pi\left(5-\frac{1}{x}\right)^2 dx$

d) $\int_{1/3}^1 \left[\left(2+\frac{1}{y}\right)^2\pi - \pi 2\right] dy$

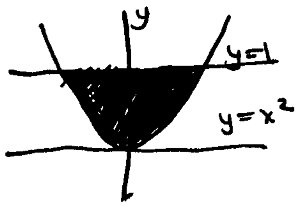
23)



$x^2+y^2=r^2$

$x^2+y^2=r^2 \quad x^2=r^2-y^2$

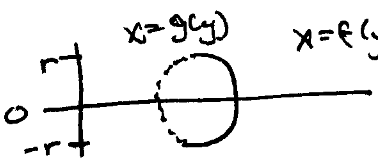
$$V = \pi \int_{r-h}^{r} (r^2-y^2)\,dy = \pi\left[r^2y - y^3/3\right]_{r-h}^{r}$$
$$= \pi \left\{\left[r^3-\frac{r^3}{3}\right] - \left[r^2(r-h) - \frac{(r-h)^3}{3}\right]\right\}$$
$$= \pi \left\{\tfrac{2}{3}r^3 - \tfrac{1}{3}(r-h)\left[3r^2-(r-h)^2\right]\right\}$$
$$= \tfrac{1}{3}\pi\left\{2r^3-(r-h)\left[3r^2-(r^2-2rh+h^2)\right]\right\}$$
$$= \cancel{\tfrac{1}{3}\pi(2r^3-2r^3-2r^2h+rh^2}$$
$$= \tfrac{1}{3}\pi\left\{2r^3-(r-h)\left[2r^2+2rh-h^2\right]\right\}$$
$$= \tfrac{1}{3}\pi\left(2r^3-2r^3-2r^2h+rh^2+2r^2h+2rh^2-h^3\right)$$
$$= \tfrac{1}{3}\pi\left(3rh^2-h^3\right) = \boxed{\tfrac{1}{3}\pi h^2(3r-h) \text{ or } \pi h^2(r-h/3)}$$



cross section of base has length $2x = 2\sqrt{y}$

square has area $A(y) = (2\sqrt{y})^2 = 4y$

$$V = \int_0^1 A(y)\,dy = \int_0^1 4y\,dy = \left[2y^2\right]_0^1 = 2$$



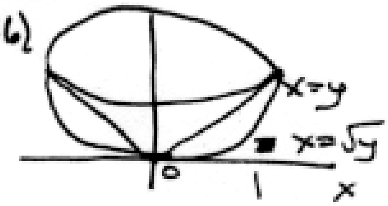
rotate circle $(x-R)^2+y=r^2$ about y-axis

a) $x = R+\sqrt{r^2-y^2} = f(y)$ right half

$x = R-\sqrt{r^2-y^2} = g(y)$ left half

$$V = \pi\int_{-r}^{r} \left\{[f(y)]^2 - [g(y)]^2\right\} dy$$
$$= 2\pi\int_0^r \left[R^2+2R\sqrt{r^2-y^2}+r^2-y^2) - (R^2-2R\sqrt{r^2-y^2}+r^2-y^2)\right]dy$$
$$= 2\pi\int_0^r 4R\sqrt{r^2-y^2}\,dy = 8\pi R\int_0^r \sqrt{r^2-y^2}\,dy$$

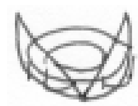
46)



$V = \int_0^1 A(y)\,dy \quad \pi\int_0^1 \left[\sqrt{y}^{\,2} - y^2\right] dy$

$= \pi\int_0^1 (y - y^2)\,dy = \pi\left[y^2/2 - y^3/3\right]_0^1 \quad \pi/6$

Cylindrical shells for volume



$\bar{x}_i = \bar{x}_i^2$ so volume =

$2\pi \bar{x}_i (\bar{x}_i - \bar{x}_i^2)\Delta x$

$V \quad \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi\, \bar{x}_i (\bar{x}_i - x_i^2)$

$\Delta x \quad \int_0 2\pi\, x(x - x^2)\, dv$

$2\pi \int_0^1 (x^2 - x^3)\,dx \quad 2\pi\left[x^3/3 \quad x^4/4\right] \quad 2\pi\left(\frac{1}{12}\right) \quad \pi/6$