Solution set #22

Math 1b

Problems: 6.3: 1,4,[redacted]21 6.4: 2,5,11 6.5: 1

extra 6.4 #16

1) $y = 2-3x \quad L = \int_{-2}^{1} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_{-2}^{1} \sqrt{1+(-3)^2}\,dx = \sqrt{10}\,[1-(-2)] = \boxed{3\sqrt{10}}$

L = distance from $(-2,8)$ to $(1,-1)$ $= \sqrt{[1-(-2)]^2+[(-1)-8]^2} = 3\sqrt{10}$

4) $y = 2^x \quad \frac{dy}{dx} = 2^x \ln 2 \Rightarrow \boxed{L = \int_0^3 \sqrt{1+(\ln 2)^2 2^{2x}}\,dx}$

21) $y = 180 - \frac{x^2}{45}$ $\quad$ $y=0$ at ground: $180 - \frac{1}{45}x^2 = 0 \quad x^2 = 45\cdot 180 \quad x = 90$

$y' =$ [redacted] $-\frac{2}{45}x \qquad L = \int_0^{90} \sqrt{1+\frac{4}{45^2}x^2}\,dx = \int_0^4 \sqrt{1+u^2}\,\frac{45}{2}\,du \quad \left[u = \frac{2}{45}x,\ du = \frac{2}{45}dx\right]$

$L = \frac{45}{2}\left[\frac{1}{2}u\sqrt{1+u^2} + \frac{1}{2}\ln(u+\sqrt{1+u^2}\right]_0^4$

$= \frac{45}{2}\left[2\sqrt{17} + \frac{1}{2}\ln(4+\sqrt{17})\right] = \boxed{209.1\,m}$

2) $g(x) = x^2 \quad [1,4]$

$g_{ave} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{2}{9}\left[x^{3/2}\right]_1^4 = \frac{2}{9}(8-1) = \boxed{14/9}$

5) a) $f_{ave} = \frac{1}{2-0}\int_0^2 (4-x^2)\,dx = \frac{1}{2}\left[4x - \frac{1}{3}x^3\right]_0^2 = \frac{1}{2}\left(8 - \frac{8}{3}\right) = \boxed{8/3}$

b) $f_{ave} = f(c) \Leftrightarrow 8/3 = 4 - c^2 \quad c^2 = 4/3 \quad \boxed{c = \frac{2}{\sqrt{3}} = 1.15}$

c)

11) $T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$

9am - 9pm $\quad t=0 = 9am \quad t = 12 = 9pm$

$$T_{ave} = \frac{1}{12-0}\int_0^{12}\left[150 + 14\sin\tfrac{1}{12}\pi t\right]dt = 1/2\left[50t - 14\cdot\tfrac{12}{\pi}\cos\tfrac{1}{12}\pi t\right]_0^{12}$$

$$= \boxed{59°F}$$

1) $$W = \int_a^b f(x)dx = \int_0^9 \frac{10}{(1+x)^2}dx = 10\int_1^{10}\frac{1}{u^2}du \quad [u = 1+x,\ du = dx]$$

$$= 10\left[\frac{-1}{u}\right]_1^{10} = \boxed{9\text{ ft-lb}}$$

Extra Credit

16) $$v_{ave} = \frac{1}{R-0}\int_0^R v(r)\,dr = \frac{1}{R}\int_0^R \frac{P}{4\eta l}(R^2 - r^2)\,dr = \frac{P}{4\eta lR}\left[R^2 r - 1/3\, r^3\right]_0^R = \frac{P}{4\eta lR}\left(\frac{2}{3}\right)R^3 =$$

$\frac{PR^3}{6\eta l}$ $\quad$ $v(r)$ is decreasing $(0, R]$ $\quad v_{max} = v(0) = \frac{PR^2}{4\eta l}$ $\quad \boxed{v_{ave} = 2/3\, v_{max}}$