

Solution set #22

Math 1b

Problems: 6.3: 1, 4, ~~21~~ 6.4: 2, 5, 11 6.5: 1
extra 6.4 #16

$$1) y = 2 - 3x \quad L = \int_{-2}^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_{-2}^1 \sqrt{1 + (-3)^2} dx = \sqrt{10} [1 - (-2)] = \boxed{3\sqrt{10}}$$

$$L = \text{distance from } (-2, 8) \text{ to } (1, -1) = \sqrt{[1 - (-2)]^2 + [(-1) - 8]^2} = 3\sqrt{10}$$

$$4) y = 2^x \quad \frac{dy}{dx} = 2^x \ln 2 \Rightarrow \boxed{L = \int_0^3 \sqrt{1 + (\ln 2)^2} 2^{2x} dx}$$

$$2) y = 180 - \frac{x^2}{45} \quad y=0 \text{ at ground: } 180 - \frac{1}{45}x^2 = 0 \quad x^2 = 45 \cdot 180 \quad x = 90$$

$$y' = \frac{-2}{45}x \quad L = \int_0^{90} \sqrt{1 + \frac{4}{45^2}x^2} dx = \int_0^4 \sqrt{1 + u^2} \frac{45}{2} du \quad [u = \frac{2}{45}x, du = \frac{2}{45}dx]$$

$$L = \frac{45}{2} \left[\frac{1}{2}u\sqrt{1+u^2} + \frac{1}{2}\ln(u + \sqrt{1+u^2}) \right]_0^4$$

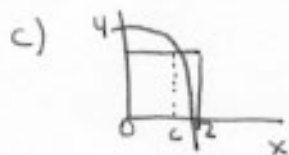
$$= \frac{45}{2} [2\sqrt{17} + \frac{1}{2}\ln(4 + \sqrt{17})] = \boxed{209.1 \text{ m}}$$

$$2) g(x) = x^2 \quad [1, 4]$$

$$g_{\text{ave}} = \frac{1}{4-1} \int_1^4 \sqrt{x} dx = \frac{1}{3} \left[\frac{2}{3}x^{3/2} \right]_1^4 = \frac{2}{9} [x^{3/2}]_1^4 = \frac{2}{9} (8-1) = \boxed{14/9}$$

$$5) a) \text{ fare} = \frac{1}{2} \int_0^2 (4-x^2) dx = \frac{1}{2} [4x - \frac{1}{3}x^3]_0^2 = \frac{1}{2} (8 - 8/3) = \boxed{8/3}$$

$$b) \text{ fare} = f(c) \Leftrightarrow 8/3 = 4 - c^2 \quad c^2 = 4/3 \quad \boxed{c = \frac{2}{\sqrt{3}} = 1.15}$$



$$11) T(t) = 50 + 14 \sin\left(\frac{\pi t}{12}\right)$$

$$9\text{am} - 9\text{pm} \quad t=0 = 9\text{am} \quad t=12 = 9\text{pm}$$

$$T_{\text{ave}} = \frac{1}{12-0} \int_0^{12} [50 + 14 \sin \frac{1}{12} \pi t] dt = \frac{1}{2} [50t - 14 \cdot \frac{12}{\pi} \cos \frac{1}{12} \pi t]_0^{12}$$

$$= \boxed{59^\circ \text{F}}$$

$$1) W = \int_a^b f(x) dx = \int_0^1 \frac{10}{(1+x)^2} dx = 10 \int_1^2 \frac{1}{u^2} du \quad [u=1+x, du=dx]$$

$$= 10 \left[\frac{-1}{u} \right]_1^2 = \boxed{9ft \cdot lb}$$

Extra Credit

$$16) V_{\text{ave}} = \frac{1}{R-0} \int_0^R v(r) dr = \frac{1}{R} \int_0^R \frac{P}{4\eta l} (R^2 - r^2) dr = \frac{P}{4\eta l R} [R^2 r - \frac{1}{3} r^3]_0^R = \frac{P}{4\eta l R} \left(\frac{2}{3}\right) R^3 =$$

$$\frac{PR^2}{6\eta l} \quad v(r) \text{ is decreasing } (0, R] \quad v_{\text{max}} = v(0) = \frac{PR^2}{4\eta l} \quad \boxed{V_{\text{ave}} = \frac{2}{3} v_{\text{max}}}$$