Review for First Exam

The structure of this review sheet is as follows: first there is a list of subjects. Next is a bunch of problems numbered according to the numbers in the first part of the sheet.

The theme of the first part of the course was that an approximate Riemann sum becomes an integral in the limit. We used this fact to answer lots of problems where we chopped things up. Here is a table to help you study for these sorts of problems:

type of problem	$\mathbf{f(x_i)\Delta x}$
1. mass from density	$f(x_i)\Delta x$ represents the mass of a slice
2. find volume	$f(x_i)\Delta x$ represents the volume of a slice
a. cylindrical shells	a. volume of shell is 2π(radius)(height)Δx
b. spherical shells	b. volume of shell is $4\pi x^2 \Delta x$
c. "washers"	c. volume of washer is $\pi\left((\mathrm{r_{outer}})^2 - (\mathrm{r_{inner}})^2\right)\Delta x$
d. other shapes	d. (area of the shape) $\cdot\Delta x$
3. find work ($= F \cdot d$)	$f(x_i)\Delta x$ represents an incremental amount of work
a. chopping up force	a. (some fn. times Δx) $\cdot$ distance
b. chopping up distance	b. (force)$\cdot\Delta x$

Here is a table of formulas for the other sorts of problems you might see:

4. average value of a function	$\frac{1}{b-a}\int_a^b f(x)dx$
5. arc length for regular functions	$\int_0^l \sqrt{1+(dy/dt)^2}dt$
6. $P(a \le X \le b)$	$\int_a^b f(t)dt$ where $f(t)$ is the probability density

You should also know how to

7. tell if something is a probability density function
8. find the median given a probability density function
9. tell if an improper integral is convergent or divergent
10. calculate convergent improper integrals
11. do u substitution, integration by parts and partial fractions

Here are some review problems. The numbers and letters refer to the subjects on the flipside.

1. A technician uses a cylinder of length l and radius r to centrifuge a blood sample. The density of the blood after the centrifuge is given by $\rho(x)$ where x is the distance from the top of the cylinder. Set up an integral representing the mass of blood in the cylinder.

2. a. A wooden top is made from wood of radially varying density. If one looks at the top from the side, holding the rotation axis vertically, it looks like a diamond of height H and width $2R$. The density of the wood is given by $\rho(x)$ where x is the distance to the **central axis**. Set up a definite integral for the mass of the top.

2. b. Someone who knows nothing about geology guesses that the density of rocks in the earth is given by e^{-x}. Write a definite integral for the mass of the earth, which we consider to be of radius R.

2. c. Consider the area enclosed by hyperbola $x^2 - y^2 = 1$, the line $y = 1 + \sqrt{2} - x$ and the line $y = x - (1 + \sqrt{2})$, where y is only allowed to range between -1 and 1. Find the volume created when this area is rotated around the y-axis.

2. d. Derive the formula for the volume of a pyramid of height H and square base of side W.

3. a. How much work does it take to build a pyramid of these dimensions if the architects use rock of density $\rho(x)$ at height x of the pyramid?

3. b. The Millennium Falcon is trapped by a tractor beam! The force of the tractor beam is $1/(e^{-x} + e^{x})$ where x is in meters and the Falcon is currently only 500 meters away from the enemy ship. How much work does Luke have to do when he uses the Force to move the Falcon to safety 5 kilometers away?

9. What if Luke wants to move the Falcon infinitely far away from the enemy ship? Will that require an infinite amount of Force? If not, compute the amount of Force required.

6. The probability density function concerning a student falling asleep in this review session is $f(x)$:

$$f(x) = \begin{cases} \frac{\pi}{4} t \sin\left(\pi (t/2)^2\right) & 0 \leq t \leq 2 \\ \text{zero} & \text{otherwise} \end{cases}$$

Check that this is actually a probability density function.

7. Find the probability that someone falls asleep in the second hour.

8. What is the median time to fall asleep?

10. What is the amount of work required to move an electron to infinity if it starts one Angstrom ($= 10^{-10}$ m) away? The electromagnetic force is a constant C times the inverse of x squared.