Problem Set #~~1~~3

#1, 3, 5, 6

1.) A sum is geometric if

$$\frac{a_n}{a_{n-1}} = r \quad \text{where } r \text{ is a constant.}$$

a) $\sum_{K=1}^{70} \left(\frac{1}{K}\right) \qquad \frac{a_n}{a_{n-1}} = \frac{\frac{1}{n}}{\frac{1}{n-1}} = \frac{n-1}{n} \neq \text{constant.}$

$\Rightarrow$ not a geometric sum

b) $\sum_{K=1}^{50} \left(\frac{1}{K}\right)^2 \qquad \frac{a_n}{a_{n-1}} = \frac{\left(\frac{1}{n}\right)^2}{\left(\frac{1}{n-1}\right)^2} = \frac{(n-1)^2}{n^2} \neq$ ~~cost~~ constant

$\Rightarrow$ not a geometric sum

c) $\sum_{K=1}^{60} \left(\frac{1}{K}\right)^K \qquad \frac{a_n}{a_{n-1}} = \frac{\left(\frac{1}{n}\right)^n}{\left(\frac{1}{n-1}\right)^{n-1}} = \frac{(n-1)^{n-1}}{n^n} \neq \text{constant}$

$\Rightarrow$ not a geometric sum

d) $\sum_{K=1}^{60} (1.01)^{\frac{K}{3}} \qquad \frac{a_n}{a_{n-1}} = \frac{(1.01)^{n/3}}{(1.01)^{n-1/3}} = \frac{(1.01)^{n/3}}{(1.01)^n (1.01)^{-1/3}} = (1.01)^{\frac{1}{3}} = r$

$$S_n = \frac{a_0(1-r^n)}{1-r} = \frac{(1.01)^{\frac{1}{3}}}{1-(1.01)^{\frac{1}{3}}}\left[1-(1.01)^{20}\right]$$

3) No. If you have ~~an~~ a series that converges and you add a finite number of terms to the beginning of the series, the series still converges. Hence, the first few terms do not affect whether the series converges.

Yes. If you have a convergent series whose sum is S. then if you add a term to the beginning of the series whose valve is a. Then, the sum of the series is S + a.

a) $\lim_{n \to \infty} a_n = 0$. <u>Must be true</u>

If $\lim_{n \to \infty} a_n = K$, then the partial sums S_n as $n \to \infty$ $\left(\lim_{n \to \infty} S_n\right)$ wouldn't equal a finite number since you would continue adding K to the partial sum as $n \to \infty$

b) $\lim_{n \to \infty} S_n = 0$. <u>False</u>

Since every term $a_K > 0$. Then, $0 < S_1 < S_2 < \ldots < S_K$ because $S_2 - S_1 = a_2 > 0$. Thus, $\lim_{n \to \infty} S_n \neq 0$.

c) Since the partial sums converge, then they converge to a finite number S. Thus, just choose $M = S+1$

<u>Must be true.</u>

$\sum_{K=5}^{\infty} a_K$ converges. <u>Must be true</u> If $\sum_{K=0}^{\infty} a_K$ converges, then $\sum_{K=5}^{\infty} a_K$ converges since the first few terms of a series doesn't affect whether a series converges.

6.) Not necessarily true. Example is $\sum_{K=1}^{\infty} \frac{1}{K}$ diverges but $\lim_{K \to \infty} \frac{1}{K} = 0$