§5.6

24. Let $u = \frac{\pi}{x}$. Then $du = -\frac{\pi}{x^2}\,dx$, so $\int \frac{\cos(\pi/x)}{x^2}\,dx = \int \cos u\left(-\frac{1}{\pi}\,du\right) = -\frac{1}{\pi}\sin u + C = -\frac{1}{\pi}\sin\frac{\pi}{x} + C$.

27. $\int \frac{e^x+1}{e^x}\,dx = \int (1+e^{-x})\,dx = x - e^{-x} + C$ [Substitute $u = -x$.]

28. Let $u = e^x + 1$. Then $du = e^x\,dx$, so $\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(e^x+1) + C$.

32. Let $u = x^2$. Then $du = 2x\,dx$, so $\int \frac{x}{1+x^4}\,dx = \int \frac{\frac{1}{2}\,du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}(x^2) + C$.

45. Let $u = x - 1$, so $u + 1 = x$ and $du = dx$. When $x = 1$, $u = 0$; when $x = 2$, $u = 1$. Thus,

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 \left(u^{3/2} + u^{1/2}\right)du = \left[\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\right]_0^1 = \tfrac{2}{5} + \tfrac{2}{3} = \tfrac{16}{15}$$

55. First write the integral as a sum of two integrals:
$\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx = \int_{-2}^{2} x\sqrt{4-x^2}\,dx + \int_{-2}^{2} 3\sqrt{4-x^2}\,dx$. The first integral is 0 by Theorem 6(b), since $f(x) = x\sqrt{4-x^2}$ is an odd function and we are integrating from $x = -2$ to $x = 2$. The second integral we interpret as three times the area of a semicircle with radius 2, so the original integral is equal to $0 + 3\cdot\frac{1}{2}(\pi\cdot 2^2) = 6\pi$.

62. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 x f(x^2)\,dx = \int_0^9 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$.

§5.7

2. $\int_0^{\pi/2}\cos^5 x\,dx = \int_0^{\pi/2}(\cos^2 x)^2\cos x\,dx = \int_0^{\pi/2}(1-\sin^2 x)^2\cos x\,dx \overset{s}{=} \int_0^1 (1-u^2)^2\,du$
$= \int_0^1 (1 - 2u^2 + u^4)\,du = \left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right]_0^1 = \left(1 - \frac{2}{3} + \frac{1}{5}\right) - 0 = \frac{8}{15}$

20 Integration HO A

a) $\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos^2 x \cos x \, dx$

Use $\cos^2 x = 1 - \sin^2 x$

and $u = \sin x$

$du = \cos x \, dx$

$$= \int u^2 (1-u^2)\, du = \int (u^2 - u^4)\, du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C = \boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}$$

b) $\int \sin^5 x \cos^4 x \, dx = \int \sin^4 x \sin x \cos^4 x \, dx$

Use $\sin^2 x = 1 - \cos^2 x$

$$= \int (1 - \cos^2 x)^2 \sin x \cos^4 x \, dx$$

$$= \int (1 - 2\cos^2 x + \cos^4 x) \sin x \cos^4 x \, dx$$

$$= \int \sin x \cos^4 x \, dx - \int 2\cos^2 x \cos^4 x \sin x \, dx + \int \cos^4 x \cos^4 x \sin x \, dx$$

let $u = \cos x$, $du = -\sin x \, dx$; let $u = \cos x$, $du = -\sin x \, dx$; let $u = \cos x$, $du = -\sin x \, dx$

$$= -\int u^4 \, du + \int 2u^6 \, du - \int u^8 \, du$$

$$= \frac{-u^5}{5} + \frac{2u^7}{7} - \frac{u^9}{9} + C = \boxed{-\frac{\cos^5 x}{5} + \frac{2\cos^7 x}{7} - \frac{\cos^9 x}{9} + C}$$