Problem Set 19 Solutions

12. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4}$ satisfies (a) of the Alternating Series Test because $\frac{1}{(n+1)^4} < \frac{1}{n^4}$ and (b) $\lim_{n\to\infty}\frac{1}{n^4} = 0$, so the series is convergent. Now $b_5 = 1/5^4 = 0.0016 > 0.001$ and $b_6 = 1/6^4 \approx 0.00077 < 0.001$, so by the Alternating Series Estimation Theorem, $n = 5$.

13. Using the Ratio Test with the series $\sum_{n=1}^{\infty}\frac{(-2)^n}{n!}$,

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|a_{n+1}\cdot\frac{1}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-2)^{n+1}}{(n+1)!}\cdot\frac{n!}{(-2)^n}\right| = \lim_{n\to\infty}\left|\frac{-2}{n+1}\right|$$

$$= 2\lim_{n\to\infty}\frac{1}{n+1} = 2(0) = 0 < 1,$$

so the series is absolutely convergent (and therefore convergent). Now $b_7 = 2^7/7! \approx 0.025 > 0.01$ and $b_8 = 2^8/8! \approx 0.006 < 0.01$, so by the Alternating Series Estimation Theorem, $n = 7$. (That is, since the 8th term is less than the desired error, we need to add the first 7 terms to get the sum to the desired accuracy.)

19. Consider the series whose terms are the absolute values of the terms of the given series. $\sum_{n=1}^{\infty}\left|\frac{(-1)^{n-1}}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$, which is a divergent p-series ($p = \frac{1}{2} \le 1$). Thus, $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$ is *not* absolutely convergent.

Series Handout (13-15)

(13) This is an alternating series. Look for the $|\text{first term}| < 10^{-8}$.

$$a_n = \left|\frac{(-.1)^n}{n!}\right| < 10^{-8}$$

Through trial and error, $\boxed{n = 7.}$ Don't forget to include the $n=0$ te

(14) Same technique as #13

$$a_n = \left|\frac{(-1)^n (.2)^{2n+1}}{(2n+1)!}\right| < 10^{-6}$$

$$= \frac{(.2)^{2n+1}}{(2n+1)!} < 10^{-6}$$

Through trial and error, $\boxed{n = 4}$ Don't forget to include the $n=0$ term

This approximation is <u>too small</u>, because it ended on a negative term and is an alternating series.

(15) a) <u>Diverges</u> $\lim_{k\to\infty} \frac{2k^2-10k}{10k^2+5k} = \frac{1}{5} \neq 0$

violates condition (iii); terms do not go to zero.

b) <u>Converges</u> $-1 \le \sin k \le 1$ $\frac{\sin k}{k^{3/2}}$ by p-test, $p = 3/2$

violates condition (ii); magnitude of terms is not decreasing.