Problem Set 5

Handout A

Problem 8 Since we are told which function bounds the area above and which function bounds it below, all that really needs to be done is the calculation of the bounds. This is done by solving the equations simultaneously.

$$\begin{aligned} x &= -x^2 + 2 \\ x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \end{aligned}$$

This implies $x = -2$ or $x = 1$. Thus, our integral will be over the interval $[-2, 1]$

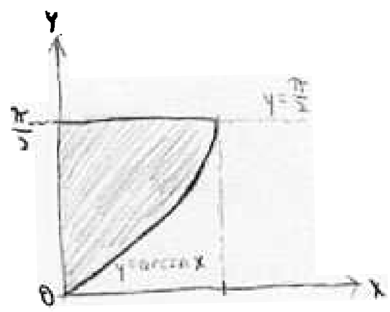
$$\int_{-2}^{1} (-x^2 + 2 - x)\, dx$$

Integration Handout

#9 The easiest way to do this is to integrate with respect to y.

$$\begin{aligned} \arcsin x &= y \\ x &= \sin y \end{aligned}$$

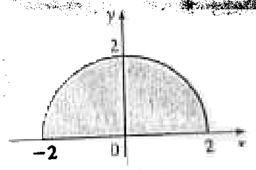
We should make sure to adjust our bounds accordingly; a quick glance at the graph shows us what we need:



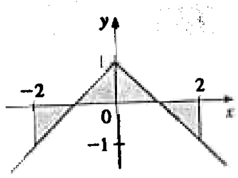
Thus, the area will be:

$$\int_0^{\pi/2} \sin y\, dy = -\cos y\Big|_0^{\pi/2} = 1$$

32. $\int_{-2}^{2} \sqrt{4-x^2}\,dx$ can be interpreted as the area under the graph of $f(x)=\sqrt{4-x^2}$ between $x=-2$ and $x=2$. This is equal to half the area of the circle with radius 2, so $\int_{-2}^{2}\sqrt{4-x^2}\,dx = \frac{1}{2}\pi\cdot 2^2 = 2\pi$.



35. $\int_{-2}^{2}(1-|x|)\,dx$ can be interpreted as the area of the middle triangle minus the areas of the outside triangles, so $\int_{-2}^{2}(1-|x|)\,dx = \frac{1}{2}\cdot 2\cdot 1 - 2\cdot\frac{1}{2}\cdot 1\cdot 1 = 0$.



2.
$$A = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1}\right)dx = \left[\tfrac{2}{3}(x+2)^{3/2} - \ln(x+1)\right]_0^2$$
$$= \left[\tfrac{2}{3}(4)^{3/2} - \ln 3\right] - \left[\tfrac{2}{3}(2)^{3/2} - \ln 1\right] = \tfrac{16}{3} - \ln 3 - \tfrac{4}{3}\sqrt{2}$$

25. $\cos x = \sin 2x = 2\sin x\cos x \Leftrightarrow 2\sin x\cos x - \cos x = 0 \Leftrightarrow \cos x\,(2\sin x - 1) = 0 \Leftrightarrow$ $2\sin x = 1$ or $\cos x = 0 \Leftrightarrow x = \frac{\pi}{6}$ or $\frac{\pi}{2}$.

$$\begin{aligned}
A &= \int_0^{\pi/6}(\cos x - \sin 2x)\,dx + \int_{\pi/6}^{\pi/2}(\sin 2x - \cos x)\,dx \\
&= \left[\sin x + \tfrac{1}{2}\cos 2x\right]_0^{\pi/6} + \left[-\tfrac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2} \\
&= \left(\tfrac{1}{2}+\tfrac{1}{2}\cdot\tfrac{1}{2}\right) - \left(0+\tfrac{1}{2}\cdot 1\right) \\
&\qquad + \left[-\tfrac{1}{2}\cdot(-1) - 1\right] - \left(-\tfrac{1}{2}\cdot\tfrac{1}{2} - \tfrac{1}{2}\right) \\
&= \tfrac{3}{4} - \tfrac{1}{2} - \tfrac{1}{2} + \tfrac{3}{4} = \tfrac{1}{2}
\end{aligned}$$