Name ______________________________

Section : ______________________________

# Math 1b
# Exam III

## December 10, 1990
## Course Head: Robin Gottlieb

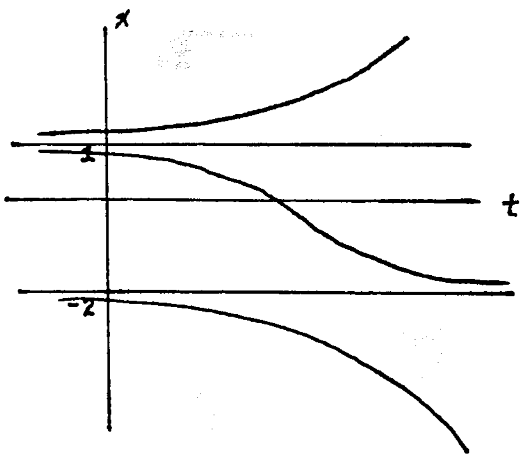
### Instructions:

1) Write your name and section leader on the lines above.
2) Write your answers in the space provided below each question.
3) You can use the backs of the test pages as scrap paper.
4) Alert us to any work that you want us to see if it's in an unusual place.
5) The multiple choice is not designed to trick you, but there will often be a few choices similar in nature to the correct answer. Circle the answer which you are sure is correct; don't just circle the first answer which seems somewhat appropriate.
6) No calculators.
7) Show all your work on the questions which are not multiple choice.
8) Think clearly, work carefully, do well.

| Part | Problem | Points | Score |
|---|---|---|---|
| I | 1 - 3 | 15 | |
| | 4 | 8 | |
| | 5 | 9 | |
| II | 1 | 9 | |
| | 2 | 9 | |
| Total | | 50 | |

**Part I:** The problems on this part of the exam are multiple choice and short answer problems. We encourage you to use your understanding of the material to answer these questions with a minimal amount of computation.

1. (5 points) Below is the graph of several particular solutions to a differential equation:




Which one of the following differential equations could have the solutions pictured above?

(a) $\dfrac{dx}{dt} = -(t-1)(t+2)$ (e) $\dfrac{dx}{dt} = x(x-1)(x+2)$

(b) $\dfrac{dx}{dt} = (t-1)(t+2)$ (f) $\dfrac{dx}{dt} = (x-1)(x+2)$

$\dfrac{dx}{dt} = (t-1)^2(t+2)$ (g) $\dfrac{dx}{dt} = (x-1)^2(x+2)$

(d) $\dfrac{dx}{dt} = (t-1)(t+2)^2$ (h) $\dfrac{dx}{dt} = (x-1)(x+2)^2$

2. (5 points) Which one of the following differential equations has a stable equilibrium at $w = 10$?

(a) $\frac{dw}{dt} = w - 10$  (e) $\frac{dw}{dt} = t - 10$

(b) $\frac{dw}{dt} = 10 - w$  (f) $\frac{dw}{dt} = 10 - t$

(c) $\frac{dw}{dt} = (w - 10)^2$  (g) $\frac{dw}{dt} = (t - 10)^2$

(d) $\frac{dw}{dt} = -(w - 10)^2$  (h) $\frac{dw}{dt} = -(t - 10)^2$

3. ( 5 points) The trajectories drawn below are solutions to which one of the following differential equations?

(a) $\frac{ds}{dt} = -s^3$

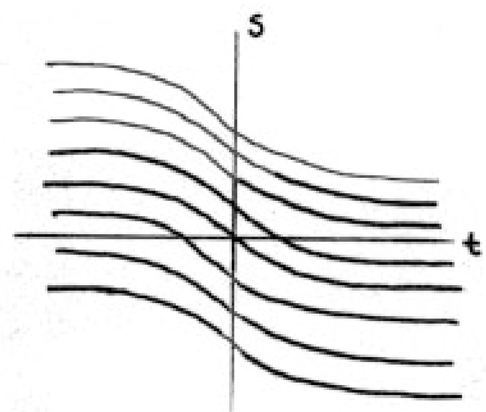
(b) $\frac{ds}{dt} = -s$

(c) $\frac{ds}{dt} = -\ e^{-t^2}$

(d) $\frac{ds}{dt} = (1 - s)s$

(e) $\frac{ds}{dt} = -t$

(f) $\frac{ds}{dt} = -3te^{t^3}$



4. (8 points) A cafeteria is preparing a large batch of hot chocolate. The chef is stirring a mixture of 4 cups hot chocolate and 1 cup warm milk in a large pot when her assistant begins to add a mixture which is 90% hot chocolate and 10% warm milk. The addition is made continuously at a rate of 2 cups per minute. As the assistant begins this addition, the mixture begins to be used at a rate of 2 cups per minute.

(a) Write a differential equation whose solution is $M(t) =$ the number of cups of milk in the pot at time $t$. Set up the differential equation but don't solve it.

Differential Equation:____________________________________________.

(b) Suppose the scenario in part (a) is changed to the following: When the assistant begins to add the mixture of 90% hot chocolate, 10% milk at the 2 cups/min. rate (and when the pot begins draining at the 2 cup/min. rate), a second assistant begins to pour in coffee at a $\frac{1}{2}$ cup/min. rate. Please write a differential equation whose solution is $M(t)$ the number of cups of milk in the pot at time $t$. Do not solve your equation.

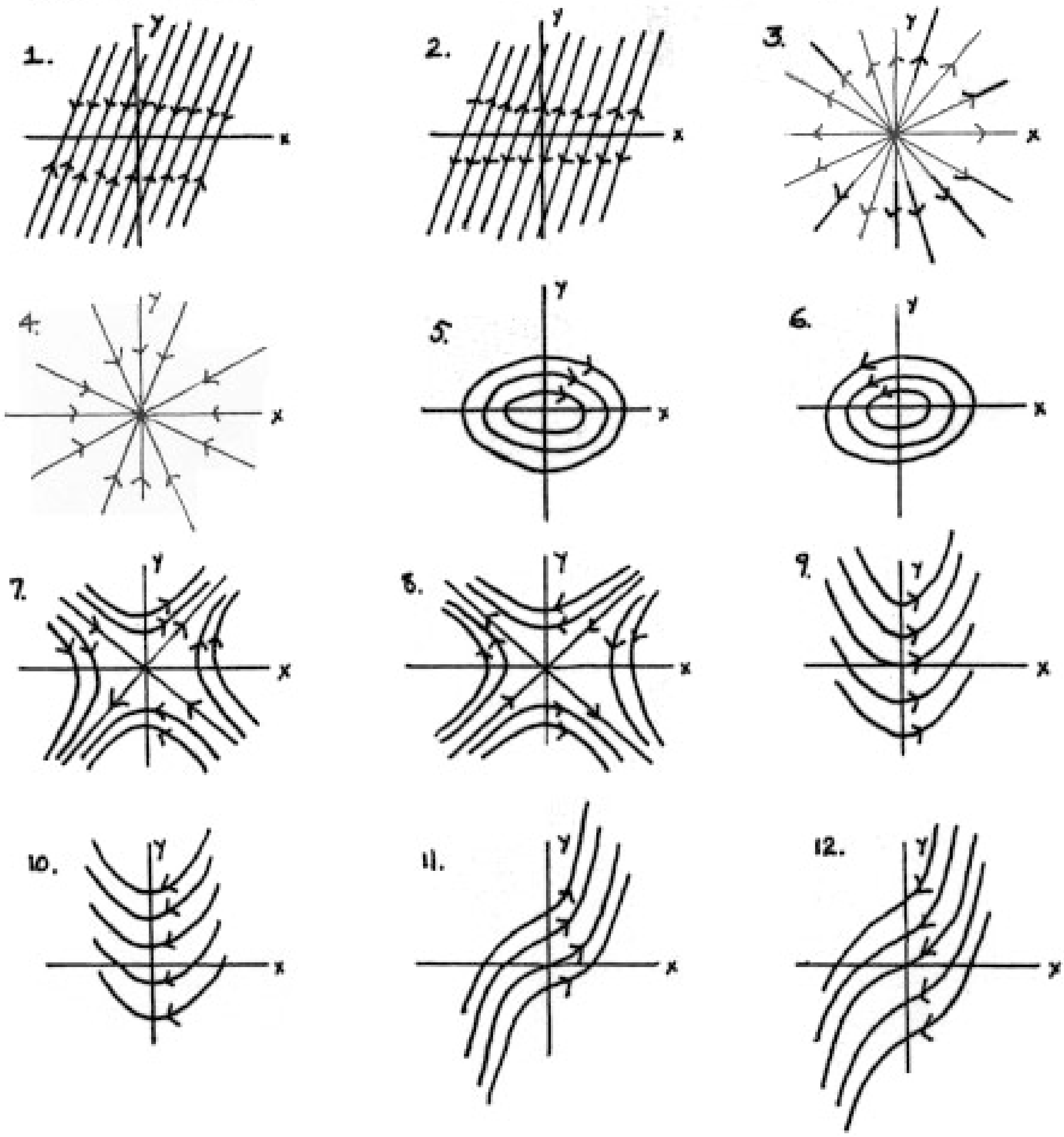
Differential Equation:____________________________________________.

5. (9 points) Consider the following systems of differential equations:

(a) $\dfrac{dx}{dt} = 1$ (b) $\dfrac{dx}{dt} = y$ (c) $\dfrac{dx}{dt} = y$

$\dfrac{dy}{dt} = x^2$ $\dfrac{dy}{dt} = -2x$ $\dfrac{dy}{dt} = 3y$

Match each of the pairs of differential equations with the correct figure below.




Answer:

Graph # corresponds to system (a).

Graph # corresponds to system (b).

Graph # corresponds to system (c).

**Part II:** You must show all your work.

1. (9 points) Since the early 1960's the population of the crown-of-thorns starfish on Australia's Great Barrier Reef has been increasing dramatically. This has been causing alarm, since it feeds on living coral. A hungry crown-of-thorns can eat its way through 5 square meters of coral in a year.

   (a) If the number of crown-of-thorns in the Barrier Reef area was 40 in 1960 and 160 in 1969, find $s(t)$, the number of crown-of-thorns starfish in the Barrier Reef area at time $t$, where $t = 0$ corresponds to the year 1960. Note: We can assume that the crown-of-thorns population increases at a rate proportional to itself. (The crown-of-thorns' only natural enemy is the triton, but the triton doesn't depend on the crown-of- thorns for food and there are very few tritons in the area so we ignore their impact.)

   (b) Assuming all crown-of-thorns starfish are hungry, how many square meters of coal have been destroyed by these starfish between 1960 and 1990?

2. (9 points) Consider the system of differential equations

$$\frac{dx}{dt} = -y$$

$$\frac{dy}{dt} = x + xy$$

(a) For what pairs $(x, y)$ is the system at equilibrium?

(b) Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{x + xy}{-y}.$$

You need not solve explicitly for $y$ or for $x$.

(c) Sketch the solution curves in the $xy$ plane "near" the point $(0,0)$. (To do this, first observe that near zero, $\frac{dy}{dt} \approx x$.) Please include arrows on your trajectories. (Do not restrict your attention to the first quadrant.)